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Transient stability analysis of power systems using Liapunov's second method

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1. INTRODUCTION

Recent blackouts have resulted in wide-scale interruptions of electrical energy supply to consumers. Because of the importance of electrical energy to modern industrial societies, these interruptions have focused attention on the problem of power system stability. The increase in the size of power systems has made it more difficult to study the performance of power systems in the transient state. Therefore the search for direct methods of determining stability has become a prominent research activity in power system analysis.

In this dissertation, the application of Liapunov's theorems on stability are considered. Inherently the method aims at obtaining information about the stability of a system without actually solving the differential equations describing its behaviour. Direct methods are very commonly used in linear systems, one of which is the well-known Routh-Hurwitz criterion. In the field of electrical power there is the well known equal area criterion for determining stability. This method, however, is limited to a two-machine system. In Liapunov's theorems systems are described by some scalar functions which behave like the physical energy of the system. From the properties of such functions system stability is analyzed. In this sense Liapunov's method is a generalization of the Lagrangian theory of equilibrium which proposes that the equilibrium point of a system occurs at the minimum, if any exists, of its potential energy. Liapunov's theorems generalize the analysis to some scalar functions satisfying certain conditions and whose properties provide information about the stability of the system. Such functions are commonly called Liapunov functions. In this dissertation, several Liapunov functions suitable for use in the study of power system dynamics are proposed.

A brief review of the various works pertinent to the present study is given in Section 2. Section 3 presents a formal statement of Liapunov's theorems. Section 4 presents a Liapunov function suitable for use in the study of power systems with negligible transfer conductances. Section 5 presents Liapunov functions which take into consideration the effect of transfer conductances. A flow diagram suitable for computer application incorporating the proposed ideas is presented in Section 6 together with an application to the combined system of the Manila Electric Company (MERALCO) and the National Power Corporation (NPC). The computer used is the IBM 360/40 digital computer of the University of the Philippines. In Section 7 are listed ideas which may be pursued as extensions of the present work. References cited in the main body of the thesis are listed in Section 8. In the method discussed in Section 6, a series solution of the swing equations during the fault condition is used. Details and proofs of convergence of the series are discussed in Appendix 1. Appendix 2 lays out the details of the MERALCO-NPC system used in the study. Appendix 3 includes the computer programs developed using Liapunov's theorems.

2. REVIEW OF LITERATURE

The necessary background on the interest in direct methods for the evaluation of stability performance of power systems is given in the introduction. In this section, a historical review of the developments up to the present is given. The key works of various authors which have set the foundations of existing knowledge on the subject are discussed chronologically.

As stated in the introduction, Lagrange's theorem on stability perhaps predates all existing direct methods of analyzing stability. Lagrange's work, expounded by Dirichlet, led to Liapunov's theorems which were developed in 1892. The publication of Liapunov's Ph.D. thesis (8) in 1949 served as an introduction of Liapunov's ideas to the West. In the late 1950's, control engineers and mathematicians found interest in Liapunov's ideas and much work on extensions and attempts of application to actual engineering systems were developed. However, such applications were not too successful at the beginning. Perhaps the first western workers in the field are Kalman and Bertram (4). Since that time considerable theoretical work has been done in the field of mathematics and control.

In the field of power, direct methods of analyzing stability have been utilized much earlier. There is the well established equal area criterion which is limited to a 2-machine system. It inspired, perhaps, Magnusson's (10) development of the concept of transient energy applied to multimachine problems. Much later, Aylett (1) developed his energy integral criterion of determining transient stability limits of power systems.

As seen from the light of present knowledge, the above methods are indeed applications of Liapunov's theorems on stability. In the 1960's power engineers started utilizing Liapunov's ideas with the publication of the works of El-Abiad and Nagappan (2) and Gless (3) who independently formulated Magnusson's and Aylett's concepts more rigorously through Liapunov's theorems. They proposed a Liapunov function which is essentially the system energy. From this function they defined the region of stability of a power system. El-Abiad and Nagappan's (2) paper considered the much accepted critical switching time as the index of stability. He proposed a method of automatically determining it by direct integration of the swing curves up to the stability limit of the post fault system. Having been introduced to the field of electrical power, Liapunov's theorems received considerable attention from power engineers and two directions of research have developed.

One direction essentially concentrated on the development of alternative Liapunov functions suitable to power system work. This is exemplified by several papers, notably those of Willems and Willems (15), Pai and Mohan (11), Luders (9), Undrill (14), Yu and Vongsuriya (16), which essentially concentrated on the development of alternative Liapunov functions suitable to power system work. The other direction is concerned with the formulation of a new index of stability using Liapunov functions. Works of Teichgraeber et al. (13) and later by Saruswati et al. (12) are of this nature. Essentially, the index developed is some kind of a normalized distance function of the instantaneous state of the system relative to the limit of stability.

This present work follows the first line of research mentioned above. It proposes a new form of Liapunov function which in certain respects is less conservative than El-Abiad's or Gless' functions. The proposed function results in a larger region of stability, and does not require the complications inherent in Willems and Willems (15) function.

Pennington, in his discussion to El-Abiad and Nagappan's paper (2), outlines the problems encountered in the application of Liapunov's ideas to actual power system problems. First is the necessity, at present, of having to integrate the system equations during the fault period. The necessity for a time solution is due to the nonautonomous nature of the problem caused by switchings in the network. Secondly, Liapunov's method requires a knowledge of the equilibrium points of the system. The equations, being nonlinear, possess multiple equilibrium points. Isolating the desired points is complicated. Third, most present functions that have been proposed are applicable only to simple systems where governor control, saturation, saliency, etc. are not considered.

The above, perhaps, are the problems that will occupy the minds of power engineers interested in the subject. Quoting Pennington, "In spite of difficulties, work must continue in order to forge new tools for analysis and control of very large power systems."

3. THEORY OF LIAPUNOV'S SECOND METHOD

Liapunov, in his "On the General Problem of Stability of Motion" (8), proposed several theorems attacking directly the problem of stability. His first method, as the Russians call it, requires the use of a series solution of a set of differential equations to investigate its stability. His second method was inspired by Dirichlet's proof of Lagrange's theorem on the stability of an equilibrium point. Essentially, what Lagrange did was to set up a potential function which has either a maximum or a minimum at a certain critical point. He then proceeded to show that, using the potential function, the trajectories either converge to the critical point, which implies stability, or they diverge, which implies instability. Liapunov generalized the method by extending the concept to general scalar functions which describe the region enclosing a given critical point. Stability is implied by the mere existence of such a function, called Liapunov function.

Before proceeding to the formal presentation of Liapunov's theorems, the following concepts and definitions are necessary for their understanding. Only autonomous systems are considered, therefore the system differential equations can be written in the form:

$$\frac{d\underline{x}}{dt} = \underline{F}\ (\underline{x}) \qquad\qquad \underline{F}(\underline{0}) = \underline{0} \tag{3.1}$$

where

1. $\underline{x} = \underline{0}$ is considered as the equilibrium point.
2. $\underline{F}(\underline{x}) \in C^1$, i.e. $\underline{F}(\underline{x})$ and its first partial derivatives are continuous.

3. all unsubscripted variables with a bar underneath are vectors.

3.1. Definitions

Liapunov used the following definition in his theory:

Stability: The origin of the system (3.1) is stable if a region containing the equilibrium point can be found such that all trajectories of the system starting within the region S remain in S ever after.

Asymptotic Stability: The system is asymptotic-stable whenever the origin (assumed to be the critical point in question) is stable as defined above and, furthermore, every solution or trajectory in S approaches $\underline{0}$ as t approaches infinity.

Instability: The equilibrium point is unstable if all the trajectories starting within the region S eventually leave the region.

3.2. Concept of Definiteness of Sign

Of importance, too, in the theorem of Liapunov is the concept of definiteness of sign. Thus, let $V = V(\underline{x})$ be a scalar function of the variables $\underline{x}$ of the system, then

1. The function $V = V(\underline{x})$ is said to be positive (negative) semi-definite if it is either positive or zero (negative or zero) in the whole region S. Mathematically,

if $V(\underline{x}) \geq 0$ for $\underline{x} \in S$

then $V(\underline{x})$ is positive semi-definite

2. The function $V(\underline{x})$ is said to be positive (negative) definite if it is positive (negative) in the whole region S except at the origin

where it vanishes. Mathematically,

$$\text{if } V(\underline{x}) > 0 \quad \text{for } \underline{x} \neq \underline{0} \qquad \underline{x} \in S$$

$$V(\underline{x}) = 0 \quad \text{for } \underline{x} = \underline{0}$$

then $V(\underline{x})$ is positive definite.

The difference between 1 and 2 is that in 1 $V(\underline{x})$ can be zero at points other than the origin.

3.3. Definition of a Liapunov Function

Let the scalar function $V = V(\underline{x})$ be continuous together with its first partial derivatives, then the derivative of $V(\underline{x})$ with respect to time exists and can be written as

$$\dot{V}(\underline{x}) = \frac{dV}{dt} = \underline{F}(\underline{x}) \cdot \text{grad } V$$

Thus let

1. $V(\underline{x})$ be positive definite for $\underline{x} \in S$
2. $\dot{V}(\underline{x})$ be negative semi-definite for $\underline{x} \in S$

then $V(\underline{x})$ is called a Liapunov function.

3.4. Liapunov's Stability Theorems

Having established the necessary background, a formal presentation of Liapunov's theorems will now be made.

Theorem I: Stability Theorem

Given the system (3.1) and if, in a region S containing the origin $\underline{x} = \underline{0}$ (or the critical point under question), a Liapunov function $V = V(\underline{x})$ exists, then the origin $\underline{x} = \underline{0}$ (or the point under question) is stable.

Theorem II: Asymptotic Stability Theorem

Given the same system (3.1) and if, in a region S containing the origin $\underline{x} = \underline{0}$ (or the critical point under question), a Liapunov function $V(\underline{x})$ exists and if $\dot{V}$ is negative definite in S then the origin $\underline{x} = \underline{0}$ (or the point under question) is asymptotically stable.

The above theorems will not be proven as such. Instead the same concept stated in a different and more convenient form for practical application will be stated and proved.

Theorem I states that the equilibrium point is stable if trajectories converge to the equilibirum point. As such, it considers certain limit cycles close to the equilibrium point as stable operation. Such oscillations in an actual system can sometimes be undesirable specially when it is desired to stay at a fixed point as much as possible. In this sense Theorem II is of more practical significance. This theorem states that trajectories starting sufficiently close to the equilibrium point will eventually end up at the equilibrium point.

The above theorems are powerful because they definitely determine the stability of a system with respect to an equilibrium point. However, they are not very well suited to actual applications wherein the system equations are nonlinear. For linear systems stability as determined from the above theorems implies absolute or complete stability. In other words, stability decided by using finite regions implies stability in the whole state space. However, for nonlinear systems the stability or instability of the system is dependent on the state of the system at any instant of time. It is in this respect that the above form of the theorems are inadequate because they give no idea concerning the "extent of stability", defined as the region which comprises the totality of all states or trajectories which are stable with respect to the equilibrium point considered.

With the above motivation, the following modified forms of Liapunov's theorems are presented as formulated in (6) together with their proofs.

Theorem III: Stability Theorem

Given the system (3.1), let a scalar Liapunov function $V(\underline{x})$ exist inside a region S_b. Let $V = b$ at the boundary of S_b and $V < b$ inside S_b. Then all states or trajectories inside S_b are stable.

Proof:

Since $V(\underline{x})$ is positive definite and nonincreasing as $t \to \infty$, then all trajectories starting inside the region S_b remain inside it ever after. This is so since the only way a trajectory can leave the region is to cross the surface $V(\underline{x}) = b$. For this trajectory to reach this state from inside the region it is necessary for $\dot{V}(\underline{x}) > 0$. But this is ruled out by the conditions imposed on $\dot{V}(\underline{x})$. Furthermore, due to the fact that $V(\underline{x})$ is positive definite and $\dot{V}(\underline{x})$ is negative semi-definite, then $V(\underline{x})$ will keep decreasing until it becomes a constant as $t \to \infty$ or when $\dot{V}(\underline{x}) \to 0$.

From the last statement of the proof of Theorem III, certain limit cycles are considered stable since the conditions on $\dot{V}(\underline{x})$ allow points other than the equilibrium point (assuming there is only one equilibrium point contained in S_b) to have $\dot{V}(\underline{x}) = 0$. For certain systems it is more desirable to exclude such operations by putting more stringent conditions on $\dot{V}(\underline{x})$ than those given in Theorem III. Thus we have the modified form of the asymptotic stability Theorem II.

Theorem IV: Asymptotic Stability Theorem

Given the system (3.1), let a Liapunov function $V(\underline{x})$ exist inside a region S_b. Also, let $\dot{V}$ be negative definite inside S_b. Let $V = b$ at the boundary of S_b and $V < b$ inside S_b. Then all states or trajectories inside S_b are asymptotically stable.

Proof:

The difference between Theorems III and IV is that $\dot{V}(\underline{x})$ is negative definite in the latter whereas it need only be negative semidefinite in the former. Thus the conditions of theorem III are satisfied and the equilibrium point is stable. Furthermore, as $t \to \infty$ the only point where $\dot{V}(\underline{x}) \to 0$ is the critical point $\underline{x} = \underline{0}$ hence all states within S_b must eventually approach $\underline{x} = \underline{0}$, which is the condition for asymptotic stability.

Seemingly simple, Liapunov's theorems entail the following problems in their application to actual systems. First is the construction of suitable Liapunov functions. Liapunov's theorems do not give specific guidelines for the setting up of such functions although a great deal of research is being done presently to make such a process more methodical. The second major problem is the fact that a Liapunov function for a particular system is not unique. Furthermore, the stability conditions derived from Liapunov's theorems are, in general, sufficient but not necessary. This implies that the conditions obtained may be too conservative, or overly rigorous, such that failure of a certain Liapunov function to demonstrate the stability of a system does not imply instability since some other function might be used successfully to demonstrate stability.

Of great importance, therefore, is the ability of a particular function to describe the extent of stability. It is desirable to find Liapunov functions which include as many stable states as possible. It is this motivation that led to this dissertation. Liapunov functions suitable for power system work are discussed in the next two sections.

4. LIAPUNOV FUNCTIONS USED IN POWER SYSTEM DYNAMICS

Before any Liapunov function can be constructed for a particular system, the system must be described by a set of differential equations. Here concern is with electrical power systems so that a mathematical model which describes the behavior of the system has to be formulated. The model, being an idealization of an actual system, entails several assumptions which have to be clearly defined.

4.1. Assumptions

Aside from the usual assumptions inherent in circuit theory, the following assumptions are used in this work:

1. The flux linkages in the system, both in the machine and in the external network, remain substantially constant. In practical situations, the flux decay is usually very much slower than the transients that occur. This assumption justifies the representation of synchronous machines as constant voltages behind their transient reactances and the external network to be in quasi-steady state.
2. Saturation and saliency of the rotating machines are neglected.
3. Governor control is much slower than the transients considered so that the mechanical input to the machines remain substantially constant.
4. Changes in speed are very small compared to synchronous speed such that the inertia constants of the synchronous machines are practically independent of speed.

5. Damping or asynchronous torque is directly proportional to the rate of change of rotor angle measured with respect to a reference frame rotating at synchronous speed.
6. Loads are represented as constant impedances.
7. Transfer conductances are negligible.

4.2. System Equations

Under the assumptions set down in Section 4.1, the behavior of the ith machine of the system can be described by the following differential equations:

$$M_i \frac{d\omega_i}{dt} + D_i\omega_i = P_{mi} - P_{ei}, \quad i = 1, 2, \ldots, N \tag{4.1}$$

$$\frac{d\delta_i}{dt} = \omega_i, \quad i = 1, 2, \ldots, N$$

and, with negligible transfer conductances, the power equations of the external network:

$$P_{ei} = E_i^2 G_{ii} + \sum_{\substack{i=1 \\ j\neq i}}^{N} E_i E_j B_{ij} \sin(\delta_i - \delta_j) \tag{4.2}$$

$$i = 1, 2, \ldots, N$$

where

t = time in secs

N = number of synchronous machines in the system

δ_i = angular displacement of rotor of the ith machine with respect to a synchronously rotating reference frame, electrical radians

$\omega_i = \frac{d\delta_i}{dt}$ = angular velocity of the rotor of ith machine with resepct to a synchronously rotating reference frame, radians/sec

s = superscript referring to stable equilibrium point

u = superscript referring to unstable equilibrium point

M_i = inertia constant of the ith machine, megajoule-seconds per megavolt ampere per electrical radian

D_i = electrical damping, p.u. power-sec/radian

P_{mi} = mechanical power input to the ith machine, p.u.

P_{ei} = electrical power output of the ith machine, p.u.

E_i = constant voltage behind transient reactance of ith machine, p.u.

G_{ii} = driving point conductance as seen through the internal bus of ith machine, p.u.

B_{ij} = transfer susceptance between the ith and the jth machines as seen through their internal buses, p.u.

4.3. Equilibrium Points of System

Having formulated the mathematical model of the system, the next step in the investigation of stability by Liapunov's method is to establish the equilibrium points of the system. These can be obtained from the system equations by setting the derivatives equal to zero. Designating the stable equilibrium angle of the ith machine by δ_i^s, then these angles are related to one another by the following equations:

$$P_{mi} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ i \neq j}}^{N} E_i E_j B_{ij} \sin (\delta_i^s - \delta_j^s) \tag{4.3}$$

$$i = 1, 2, \ldots, N$$

There are N equations and N unknowns, but a further check shows that the rank of the system is (N-1). This is due to the fact the equations depend only on the difference of the angles and not on their actual values. To bypass this complication, a reference angle is chosen which is maintained at some particular value and (N-1) of the equations are solved for the rest of the angles. It is common practice to choose the reference machine to be the one with the largest interia in the system. This practice is followed in this work. It must be noted that this is convenient and presents many advantages as far as calculations are concerned but it is not necessary.

Having set a reference machine, Equation 4.3, which is nonlinear due to the presence of the sine terms, still possesses an infinite number of solutions. However, the region of interest will be limited to that which contains only one stable equilibrium point.

4.4. Construction of a Suitable Liapunov Function

Due to Equations 4.3, the system Equations 4.1 can now be written in the form

$$(4.4) \qquad M_i \frac{d\omega_i}{dt} + D_i\omega_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} E_iE_jB_{ij} \left[\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)\right]$$

$$\frac{d\delta_i}{dt} = \omega_i \qquad\qquad i = 1, 2, \ldots, N$$

which is a more convenient form for the mathematical manipulations to follow.

At this point it is helpful to restate the properties of a scalar function which make it suitable for predicting system stability:

1. The function must be positive definite in some region surrounding origin.
2. The time derivative of the function along the trajectories of the system which lie inside the region must be negative definite,

or at least be negative semi-definite.

Inspired by La Salle and Lefschetz's (6) manipulation of Lienard's equations in two dimensions, the following change of variables is introduced into the system equations. Let

$$y_i = \omega_i + k_i \, (\delta_i - \delta_i^s), \qquad i = 1, 2, \ldots, N \tag{4.5}$$

Under this transformation, Equations 4.4 become

$$M_i \frac{dy_i}{dt} + (D_i - k_i M_i) \frac{d\delta_i}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^{N} E_i E_j B_{ij} \times [\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)] \tag{4.6}$$

$$\frac{d\delta_i}{dt} = y_i - k_i \, (\delta_i - \delta_i^s) \qquad i = 1, 2, \ldots, N$$

Using Lienard's variables, $\dot{V}$ may now be defined to be

$$\frac{dV}{dt} = \sum_{i=1}^{N} \{ M_i y_i \frac{dy_i}{dt} + k_i \, (D_i - k_i M_i)(\delta_i - \delta_i^s) \, \omega_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} E_i E_j B_{ij} \, [\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)] \, \omega_i \} \tag{4.7}$$

which, in view of Equation 4.6, is equivalent to

$$\frac{dV}{dt} = -\sum_{i=1}^{N} (D_i - k_i M_i) \, \omega_i^2 + \sum_{i=1}^{N} k_i \sum_{\substack{j=1 \\ j \neq i}}^{N} E_i E_j B_{ij} \times [\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)] \, (\delta_i - \delta_i^s) \tag{4.8}$$

Noting that $B_{ij} = B_{ji}$, $\sin(\delta_i - \delta_j) = -\sin(\delta_j - \delta_i)$, Equations 4.7 and 4.8 become

$$\frac{dV}{dt} = \sum_{i=1}^{N} [(M_i y_i \frac{dy_i}{dt} \div k_i \, (D_i - k_i M_i)(\delta_i - \delta_i^s) \, \omega_i] - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_i E_j B_{ij} \, [\sin(\delta_i - \delta_i^s) - \sin(\delta_i - \delta_j)] \, (\omega_i - \omega_j) \tag{4.9}$$

$$(4.10)\qquad \frac{dV}{dt} = - \sum_{i=1}^{N} (D_i - k_i M_i)\,\omega_i^{\,2} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_i E_j B_{ij}$$

$$\times[\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)][k_i\,(\delta_i - \delta_i^s) - k_j(\delta_j - \delta_j^s)]$$

Desiring V to be a Liapunov function of the system, its time derivative as given by either Equations 4.9 or 4.10 must satisfy condition 2 given at the start of this section. Examining Equation 4.10, the first summation of terms

$$(4.11)\qquad \sum_{i=1}^{N} (D_i - k_i M_i)\,\omega_i^{\,2} \ge 0$$

if

$$(4.12)\qquad k_i \le \frac{D_i}{M_i}\,, \qquad\qquad i = 1, 2, \ldots, N$$

The terms under the double summation sign are all negative in some region if

$$(4.13)\qquad k_i = k_j \equiv k \ge 0 \qquad\qquad i, j = 1, 2, \ldots, N$$

that is, all the k_i's are equal to some constant k together with the condition

$$(4.14)\qquad |\delta_i^s - \delta_j^s| < \frac{\pi}{2}$$

Inequality 4.14 is nothing more than a sufficient condition for the equilibrium point under consideration to be steady state stable. This is also imposed by El-Abiad and Nagappan in their work (2).

Combining Equations 4.12 and 4.13, the condition on k (the constant value of all the k_i's), become

$$(4.15)\qquad 0 \le k \le \min_{i=1,\,N} \left\{\frac{D_i}{M_i}\right\}$$

Equation 4.10 now becomes

$$(4.16)\qquad \frac{dV}{dt} = -\sum_{i=1}^{N} (D_i - k_i M_i)\,\omega_i^2 + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} E_i E_j B_{ij}\,k$$

$$\times[(\delta_i - \delta_j) - (\delta_i^s - \delta_j^s)][\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)]$$

which, under the conditions stated, is negative definite inside some region, still undefined, around the equilibrium point.

In the form of Equation 4.9, $\frac{dV}{dt}$ is in an integrable form noting that

$$k_i = k$$

$$\frac{d}{dt}(\delta_i - \delta_i^s) = \omega_i$$

t is eliminated to obtain a closed form of the solution. This results into the following equation:

$$(4.17)\qquad V(\underline{\delta},\underline{\omega}) = \sum_{i=1}^{N} \frac{1}{2} M_i y_i^2 + \sum_{i=1}^{N} \frac{1}{2} k\,(D_i - kM_i)(\delta_i - \delta_i^s)^2$$
$$- \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} E_i E_j B_{ij}\,[(\delta_i - \delta_i^s - \delta_j + \delta_j^s)$$
$$\times \sin(\delta_i^s - \delta_j^s) + \cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s)]$$

or due to Equation 4.3

$$(4.18)\qquad V(\underline{\delta},\underline{\omega}) = \sum_{i=1}^{N} \frac{1}{2} M_i y_i^2 + \sum_{i=1}^{N} \frac{1}{2} k\,(D_i - kM_i)(\delta_i - \delta_i^s)^2$$
$$- \sum_{j=1}^{N} (P_{mi} - E_i^2\,G_{ii})(\delta_i - \delta_i^s)^2$$
$$- \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} E_i E_j B_{ij}\,[\cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s)]$$

From Equation 4.18 the first two summation of terms are obviously positive definite and the remaining terms under the double summation are the result of the integration of the following

$$(4.19)\qquad \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\int_{(\delta_i^s-\delta_j^s)}^{(\delta_i-\delta_j)} E_iE_jB_{ij}\,[\sin(\delta_i^s-\delta_j^s) - \sin(\delta_i-\delta_j)]\,d(\delta_i-\delta_j)$$

Each of the integral terms of Equation 4.19 are positive definite under the same conditions imposed in $\frac{dV}{dt} \leq 0$.

Having established that the function $V(\underline{\delta},\underline{\omega})$ is a proper Liapunov function, the next step is to identify the region in which the conditions of stability are satisfied. For obvious practical reasons, it is desired that such a region be the biggest possible that can be obtained using the function defined. From Theorem IV the region is defined to be

$$(4.20)\qquad S_b:\ V(\underline{\delta},\underline{\omega}) < b$$

Noting that Equation 4.18 or 4.19 contain periodic functions, then in the whole state space, V contains relative maximums. The biggest region, therefore, where Equation 4.20 is satisfied is that where the surface $V(\underline{\delta},\underline{\omega}) = b$ passes through the "closest" relative maximum of V, where by "closest" is meant to be that with the smallest value of V. From the system equations, the points of relative maximum occur at the solutions $\underline{\delta}^u$ of

$$(4.21)\qquad \sum_{\substack{j=1\\ j\neq i}}^{N} E_iE_jB_{ij}\,[\sin(\delta_i^u-\delta_j^u) - \sin(\delta_i^s-\delta_j^s)] = 0 \qquad i = 1, 2,\ldots, N$$

or in more familiar form

$$(4.22)\qquad P_{mi} - E_i^2\,G_{ii} - \sum_{\substack{j=1\\ j\neq i}}^{N} E_iE_jB_{ij}\,\sin(\delta_i^u-\delta_j^u) = 0 \qquad i = 1, 2,\ldots, N$$

which is nothing more than the steady state power equation. Points of relative maximums, therefore, occur at the unstable equilibrium points of the system whereas points of relative minimums occur at the stable equilibrium points.

The necessity of having to know the equilibrium points of the system is one of the big drawbacks of Liapunov's method. Equation 4.22, being nonlinear, requires an iterative technique to obtain its solutions. Furthermore, the system possesses an infinite set of solutions so that a knowledge of the approximate location of the desired root must be known for any numerical process to converge to it. An ingenuous approach is used by El-Abiad which from physical reasoning searches for the machine most likely to go unstable, and such knowledge is used to initialize the iteration process. Mathematically, it is difficult to prove that such a process converges to the desired root but physical reasoning based on experience with power system dynamics justifies it.

With the assumption that the "closest" unstable point is available, the region S_b: $V < b$ is the largest region which satisfies the conditions of Liapunov's theorems for a given set of system parameters. However, there is still the constant k in Equation 4.18 which can take a range of values as given in Equation 4.15 and is reproduced below:

$$(4.15) \qquad 0 \le k \le D_M$$

$$D_M = \min_{i=1,\,N} \left\{\frac{D_i}{M_i}\right\}$$

Again, the constant k must be chosen to optimize the description of the extent of stability by the given function.

If $\frac{dV}{dt} = 0$ in the whole of state space, then V = constant describes the actual trajectories of the system. Hence, the smaller the value of $\frac{dV}{dt}$, the closer the possibility that the V = constant surfaces approximate the trajectories of the system. A suitable criterion, therefore, of the ability of a Liapunov function to describe the extent of stability is the magnitude of $\frac{dV}{dt}$. The constant k must be chosen to minimize $\frac{dV}{dt}$. With k = 0, the function becomes

$$(4.23)\qquad V = \sum_{i=1}^{N} \frac{1}{2} M_i \omega_i^2 - \sum_{i=1}^{N} (P_{mi} - E_i^2 G_{ii})(\delta_i - \delta_i^s)$$

$$- \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_i E_j B_{ij} \left[\cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s)\right]$$

$$\frac{dV}{dt} = - \sum_{i=1}^{N} D_i \omega_i^2$$

which is exactly the Liapunov function proposed by El-Abiad and Nagappan (2) and Gless (3).

4.5. Application to a One-Machine System

To illustrate the effect of k on the extent of stability as described by the given function, the above concept is applied to a machine connected to an infinite bus with damping exaggerated to magnify the differences.

For a one machine system,

$$(4.24)\qquad M \frac{d\omega}{dt} + D\omega = P_m - P_e \sin \delta$$

$$\frac{d\delta}{dt} = \omega$$

Let the subscript 1 refer to choice of k = 0 and the subscript 2 to choice of $k = \frac{D}{M}$. Then

$$(4.25)\qquad V_1 = \frac{1}{2} M\omega^2 - P_m(\delta - \delta^s) + P_e \cos \delta^s - P_e \cos \delta$$

$$V_2 = \frac{1}{2} My^2 - P_m(\delta - \delta^s) + P_e \cos \delta^s - P_e \cos \delta$$

and using the following numerical values

$$H = 3.77$$

$$M = \frac{2H}{\omega_o} = .02$$

$$D = .05$$

$$P_m = 1$$

$$P_e = \sqrt{2}$$

$$\delta^s = 45^o$$

$$\delta^u = 135^o$$

Equation 4.18 becomes,

$$V_1 = .01 \times \omega^2 - (\delta - \frac{\pi}{4}) + 1 - \sqrt{2} \cos \delta$$

$$V_2 = .01 \times y^2 - (\delta - \frac{\pi}{4}) + 1 - \sqrt{2} \cos \delta$$

and

$$b_1 = \frac{-\pi}{2} + 1 - \sqrt{2} \cos \left(\frac{3\pi}{4}\right)$$

$$b_2 = .01 \times \left(\frac{2.5 \times \pi}{2}\right)^2 - \frac{\pi}{2} + 1 - \sqrt{2} \cos \frac{3\pi}{4} .$$

Then the regions

$$V_1 < b_1$$

$$V_2 < b_2$$

are computed and shown in Figure 4.1.

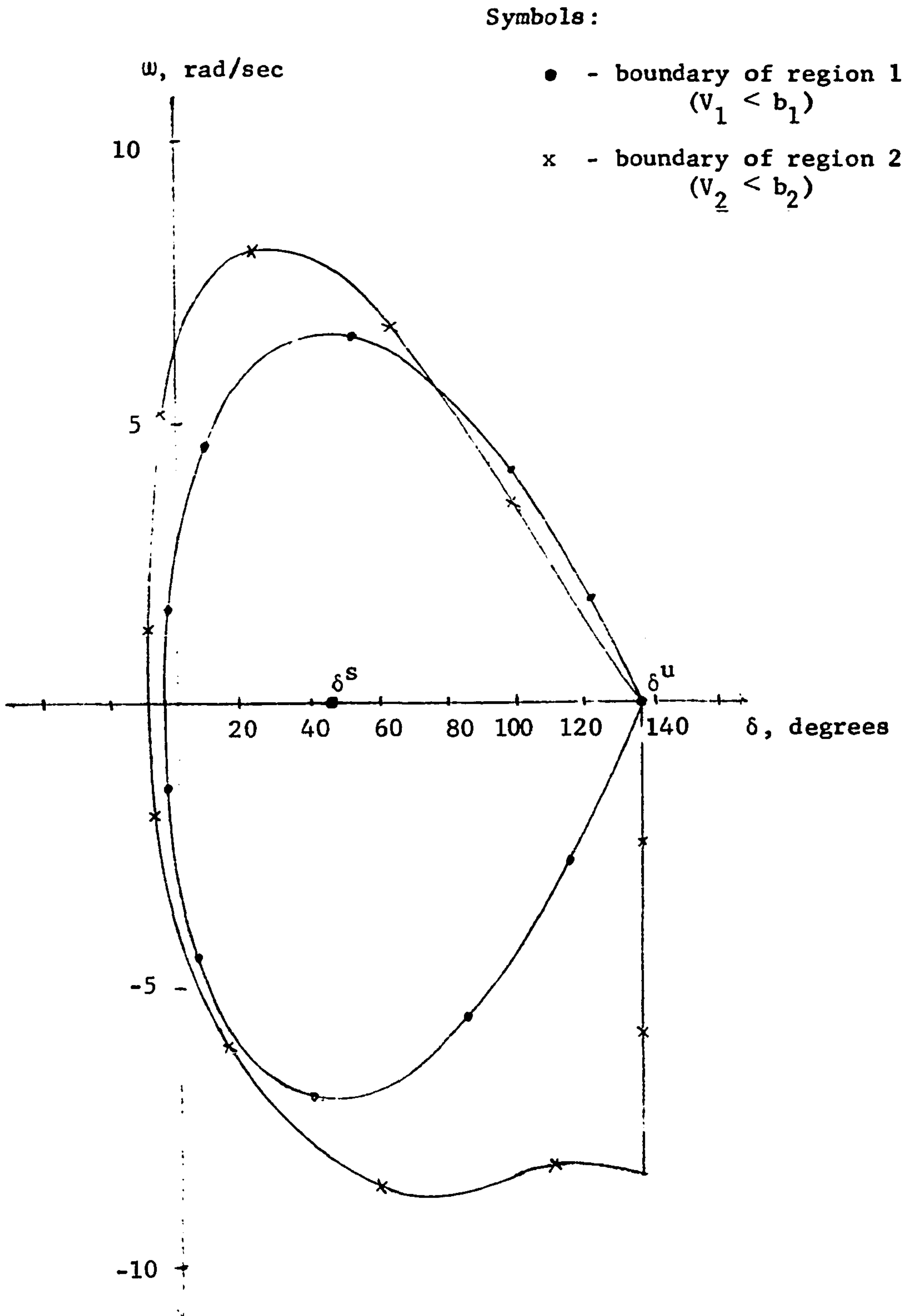

Figure 4.1. Regions of stability of a one machine system

5. ON THE EFFECT OF TRANSFER CONDUCTANCES

In Section 4, it was assumed that the effect of the transfer conductances of the system is negligible and, if present, results in a conservative estimate of the critical clearing time. The results of the study (Case II) given in Section 6.2 verify the latter effect. However, the estimate obtained is too conservative for any power system having appreciable transfer conductances. For such systems, therefore, the transfer conductances have to be accounted for in some way in the Liapunov function. This section is devoted to an analysis of the effect of transfer conductances.

5.1. System Equations Including Transfer Conductances

Since interest is only in the effect of transfer conductances, damping will be neglected so that the equations describing the system become

$$M_i \frac{d\omega_i}{dt} = P_{mi} - E_i^2 G_{ii} - \sum_{\substack{j=1 \\ j\neq i}}^{N} [E_i E_j B_{ij} \sin(\delta_i - \delta_j) + E_i E_j G_{ij} \cos(\delta_i - \delta_j)] \tag{5.1}$$

$$\frac{d\delta_i}{dt} = \omega_i \qquad i = 1, 2, \ldots, N$$

G_{ii} - driving point conductance of bus i

G_{ij} - transfer conductances between bus i and j and all other symbols as defined before.

As before, the equilibrium points of the system are defined by setting the system equations equal to zero. Thus,

$$(5.2)\qquad P_{mi} = E_i^2 G_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{N} [E_iE_jB_{ij} \sin(\delta_i^s - \delta_j^s) + E_iE_jG_{ij} \cos(\delta_i^s - \delta_j^s)] \qquad i = 1, 2, \ldots, N$$

The system Equations 5.1 may now be written into the following alternate form.

$$(5.3)\qquad M_i \frac{d\omega_i}{dt} = \sum_{\substack{j=1\\ i\neq j}}^{N} \{E_iE_jB_{ij}[\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)] + E_iE_jG_{ij}[\cos(\delta_i^s - \delta_j^s) - \cos(\delta_i - \delta_j)]\}$$

$$\frac{d\delta_i}{dt} = \omega_i \qquad i = 1, 2, \ldots, N$$

5.2. Liapunov Functions which Include the Effect of Transfer Conductances

Following the construction done in Section 4, it is logical to set the Liapunov function into the form

$$(5.4)\qquad V = \int_0^{\omega_i} M_i\omega_i d\omega_i - \int_{\delta_i^s}^{\delta_i} \sum_{\substack{j=1\\ j\neq i}}^{N} E_iE_jB_{ij}[\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)]\, d\delta_i - \sum_{i=1}^{N} \int_{\delta_i^s}^{\delta_i} \sum_{\substack{j=1\\ j\neq i}}^{N} E_iE_jG_{ij} \times[\cos(\delta_i^s - \delta_j^s) - \cos(\delta_i - \delta_j)]\, d\delta_i$$

Difficulty, however, arises due to the nonconservative nature of the terms involving G_{ij}. That is, the value of the last group of integrals above depends upon the path of integration. This is expected since these terms are associated with losses and hence give rise to energy that is unrecoverable. The path of integration therefore should coincide with actual trajectories of the system and hence must satisfy Equations 5.1. But knowledge of the time solutions of the system equations is exactly what the method is aiming to bypass. Thus the effect of the transfer conductances have to be accounted for in some way in the Liapunov function to be used.

A function suggested by El-Abiad is the following:

$$V = \sum_{i=1}^{N} \frac{1}{2} M_i \omega_i^2 + \sum_{i=1}^{N} (E_i^2 G_{ij} - P_{mi})(\delta_i - \delta_i^s)^2 \tag{5.5}$$
$$+ \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \{E_i E_j B_{ij} [\cos (\delta_i^s - \delta_j^s)$$
$$- \cos (\delta_i - \delta_j)] + E_i E_j G_{ij} [\sin (\delta_i^s - \delta_j^s) \ - \sin (\delta_i - \delta_j)]\}$$

or in an alternative form, in view of Equation 5.2

$$V = \sum_{i=1}^{N} \frac{1}{2} M_i \omega_i^2 - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_i E_j B_{ij} [\sin (\delta_i^s - \delta_j^s) \tag{5.6}$$
$$\times (\delta_i - \delta_j - \delta_i^s + \delta_j^s) + \cos (\delta_i^s - \delta_j^s) - \cos (\delta_i - \delta_j)$$
$$- \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_i E_j G_{ij} \{\cos (\delta_i^s - \delta_j^s) [(\delta_i + \delta_j) - (\delta_i^s + \delta_j^s)]$$
$$+ \sin (\delta_i^s - \delta_j^s) - \sin (\delta_i - \delta_j)\}$$

Although this function accounts for the transfer conductances, it poses a complication in practical applications. The difficulty of applying it stems from the fact that it depends on both the sum and the differences of angles. The angle differences are unique for a given post-fault system while the angle sums are not; they depend on the actual trajectories. This implies that shifting all the δ_i^s angles by some constant changes the value of V at a given point in state space although the differences $(\delta_i^s - \delta_j^s)$ are maintained constant. The appropriate value to use is, of course, the actual point $\underline{\delta}^s$ at which the system will eventually approach equilibrium. However, knowledge of the exact point cannot be obtained from the system equations. The same arguments apply when the index of stability $b = V(\underline{\omega}^u, \underline{\delta}^u)$ is obtained, where (ω^u, δ^u) is the closest unstable equilibrium point. By replacing δ_i by δ_i^u in Equation 5.6, the index b becomes a function of the sums of angles of both the unstable and stable points. Again, knowledge of the exact point $\underline{\delta}^u$ is required which cannot be derived from the system equations.

However, for certain electrical systems, the above function may be practical. For the system considered by El-Abiad, one of the machines represented had a moment of inertia much larger than the rest of the machines. The steady state angle of such a machine in the post-fault system would be very close to its prefault value. Therefore the choice of the machine with the largest inertia as reference machine in the determination of $\underline{\delta}^s$ and $\underline{\delta}^u$ results in these points being close enough to their actual post fault value.

However, for machines with moments of inertia of the same order of magnitude, and assuming that the post-fault system is very different from the prefault system, and if the fault is severe enough, then in general, all the machines will be displaced. The system equations do not provide a clear idea as to the exact point $\underline{\delta}^s$ to which the system will settle.

For such systems, a possible alternate way of accounting for the transfer conductances will now be developed. Adding all N Equations of 5.3 results in

$$(5.7) \qquad \sum_{i=1}^{N} M_i \frac{d\omega_i}{dt} = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} E_i E_j G_{ij}[\cos \delta_{ij}^s - \cos \delta_{ij}]$$

which is interesting in the sense that it is a function only of the transfer conductances of the system.

Now we note that if the transfer conductances of the system are zero, then

(5.8) $$\sum_{i=1}^{N} M_i \frac{d\omega_i}{dt} = 0$$

which implies that $\sum_{i=1}^{N} M_i \omega_i$ = constant, along the system trajectory. For stable trajectories therefore where $\omega_i^s = 0$,

(5.9) $$\sum_{i=1}^{N} M_i \omega_i = 0$$

Thus

(5.10) $$\sum_{i=1}^{N} M_i \delta_i = \sum_{i=1}^{N} M_i \delta_i^s = \text{constant}$$

The above result implies that knowing any point in a stable trajectory, the exact point $\underline{\delta}^s$ can be determined from the above equation together with the difference equations determined from the steady state equations. This is of no practical significance in this work, however, since for a system with negligible transfer conductances only the differences of angles are necessary. This is so since the Liapunov function chosen in Section 4 is expressed in terms of angle differences only.

Equation 5.10 can be interpreted differently if we define

(5.11) $$\overline{\delta} = \frac{1}{M} \sum_{i=1}^{N} M_i \delta_i \qquad M = \sum_{i=1}^{N} M_i$$

$$\overline{\omega} = \frac{d\overline{\delta}}{dt} = \frac{1}{M} \sum_{i=1}^{N} M_i \omega_i$$

We note that $\overline{\delta}$ is some kind of mean angle and $\overline{\omega}$ as some mean velocity. Such a point has been aptly called the "inertial center" by Luders (9). Now for the lossless case where the transfer conductances are zero.

$$\text{(5.12)} \qquad \bar{\delta} = \text{constant}$$
$$\bar{\omega} = 0$$

at all instants of time, or at all points of the system trajectory. Hence, for the lossless case the inertial center is fixed.

For the case where transfer conductances are present

$$\text{(5.13)} \qquad M\frac{d\bar{\omega}}{dt} = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} E_iE_jG_{ij}[\cos(\delta_i^s - \delta_j^s) - \cos(\delta_i - \delta_j)]$$

which implies that for a system with losses, the inertial center moves. The rate of change of the kinetic energy of this inertial center is given by Equation 5.13. The right hand side of 5.13 gives the rate of energy dissipation in the transfer conductances of the network. Therefore, this energy represents the energy used in moving the inertial center. Since the movement of this center does not affect the system stability, it should be accounted for in the formulation of the Liapunov function.

From the arguments presented above, the following Liapunov function is chosen.

$$\text{(5.14)} \qquad V = \sum_{i=1}^{N} \frac{1}{2}M_i\omega_i^2 - \frac{1}{2}M\bar{\omega}^2 - \sum_{i=1}^{N} \sum_{j=i+1}^{N} E_iE_jB_{ij}$$
$$\times[\sin(\delta_i - \delta_j^s)(\delta_i - \delta_j - \delta_i^s + \delta_j^s) + \cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s)]$$

The above is equivalent to

$$\text{(5.15)} \qquad V = \sum_{i=1}^{N} \frac{1}{2}M_i(\omega_i - \bar{\omega})^2 - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_iE_jB_{ij}$$
$$\times[\sin(\delta_i^s - \delta_j^s)(\delta_i - \delta_j - \delta_i^s + \delta_j^s) + \cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s)]$$

since

$$\sum_{i=1}^{N} \frac{1}{2}M_i(\omega_i - \overline{\omega})^2 = \sum_{i=1}^{N} \frac{1}{2}M_i[\omega_i^2 - 2\omega_i\overline{\omega} + \overline{\omega}^2]$$

$$= \sum_{i=1}^{N} \frac{1}{2}M_i[\omega_i^2 - 2\omega_i\overline{\omega} + 2\overline{\omega}^2 - \overline{\omega}^2]$$

$$= \sum_{i=1}^{N} \frac{1}{2}M_i\omega_i^2 - \frac{1}{2}M\overline{\omega}^2 - 2\overline{\omega}\ (\sum_{i=1}^{N} M_i\omega_i - M\overline{\omega})$$

$$= \sum_{i=1}^{N} \frac{1}{2}M_i\omega_i^2 - \frac{1}{2}M\overline{\omega}^2$$

Thus such a V function is positive definite.

In the form of Equation 5.15, V can be interpreted to be the total energy of the system with speeds or angles measured with respect to the inertial reference. This is logical since, as has been stated, the velocity or location of the inertial center does not really affect the stability properties of the system.

A different formulation, more mathematical in nature, leads to the same result. It has been assumed so far that the steady point $\underline{\delta}^s$ is fixed in state space as defined by the difference equations $\delta_i^s - \delta_j^s = \delta_{ij}^s$. The mathematics describing the system would still be valid if the stable equilibrium point is allowed to move in time such that some kind of energy is associated with it, which, although part of the system total energy, does not affect its stability. Thus letting $\underline{\omega}^s$ be the velocity associated with $\underline{\delta}^s$ and to maintain the difference $\delta_i^s - \delta_j^s$ constant, then all such speeds are equal, thus $\omega_i^s = \omega_s$. The the Liapunov function becomes

$$V = \sum_{i=1}^{N} \int_{\omega_s}^{\omega_i} M_i\omega_i d\omega_i - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \int_{\delta_{ij}^s}^{\delta_{ij}} E_iE_jB_{ij}$$

$$\times[\sin(\delta_i^s - \delta_j^s) - \sin(\delta_i - \delta_j)]\ d(\delta_i - \delta_j)$$

$$= \sum_{i=1}^{N} \frac{1}{2}M_i\omega_i^2 - \frac{1}{2}M\omega_s^2 - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_iE_jB_{ij}$$

$$\times[\sin(\delta_i^s - \delta_j^s)(\delta_i - \delta_j - \delta_j^s - \delta_j^s) - \cos(\delta_i - \delta_j) + \cos(\delta_i^s - \delta_j^s)]$$

So far ω_s has been arbitrarily defined. However, by choosing $\omega_s = \bar{\omega}$, the velocity of the inertial center, the V function becomes positive definite. Hence as before the V function is in the form of Equation 5.15.

5.3 Region of Stability

As in Section 4, the extent of stability is defined as $V < b$ where b is the closest, smallest relative maximum of V. It has been shown in Section 4 that the closest relative maximum of V occurs at the closest unstable equilibrium point $(\underline{\omega}^u, \underline{\delta}^u)$ of the system. Thus define b to be

$$b = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_iE_jB_{ij}[\sin\delta_{ij}^s(\delta_{ij} - \delta_{ij}^s)$$

$$+ \cos\delta_{ij}^u - \cos\delta_{ij}^s] \leq \sum_{i=1}^{N} \frac{1}{2}M_i(\omega_i^u - \bar{\omega}^2)$$

$$- \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} E_iE_jB_{ij}[\sin\delta_{ij}^s(\delta_{ij}^u - \delta_{ij}^s) + \cos\delta_{ij}^u - \cos\delta_{ij}^s]$$

which again results in a conservative description of the region of stability, but which perhaps is less conservative than the region described by the function of Section 4.

6. APPLICATION OF LIAPUNOV'S THEOREMS TO TRANSIENT STABILITY PROBLEMS

A mathematical model together with suitable Liapunov functions to describe a power system has been formulated in Sections 4 and 5. A numerical method utilizing the model together with Liapunov's theorems will now be presented which will automatically determine the critical clearing time of a power system due to transient disturbances.

A method used by most researchers is to integrate point-by-point the swing equations from the instant of fault until the state of the system reaches the boundary of stability of the post-fault system. This, of course, assumes that the initial conditions of the network are available as obtained from a load flow study.

In this dissertation a novelty in the method is introduced wherein a series solution is used for the fault period and an iteration scheme based on the bisection method is used to obtain the critical switching time. Details of the series used are discussed in Appendix 1. The use of a series solution was motivated by the fact that the equations describing each machine are of standard type. Furthermore, for transient stability studies the duration of fault is usually short enough. During this time the machines are fairly well behaved mathematically so that a time series solution is suitable. The above ideas, together with previous concepts introduced, are unified into a computer program to automatically determine the critical switching time.

6.1. Computer Flow Chart

Figure 6.1 shows the basic steps taken to undertake a transient stability study of a power system using Liapunov's theorems. Each step will now be discussed individually.

Step 1: Load Flow Study. This is necessary to determine the initial conditions of system.

Step 2: Determination of Equilibrium Points. The post-fault steady state equations are solved to obtain the stable and "closest" unstable equilibrium points of the system. This step is necessary since Liapunov's theorems require a knowledge of these points. The system equations which are nonlinear are solved by an iterative scheme. In this work, a modified Newton-Raphson method is used. The solution to which the method converges depends on the initial values used to start the iteration. For most problems, the stable equilibrium state of the post-fault system is usually close to the prefault state so that the latter can be used to initiate the iteration. As for the "closest" unstable point, El Abiad proposed an ingenious method whereby the smallest relative maximum occurs in the vicinity of the point $\delta_i = \delta_i^s$ except for machine m where $\delta_m = \pi - \delta_m^s$. Machine m is defined to be the machine most likely to go unstable. It is most probably the machine with the largest initial acceleration. The above estimate of angles are then used to solve the post-fault steady state equations.

Step 3: Calculation of the Limit of Stability. With $\underline{\delta}^u$ and $\underline{\delta}^s$ available, the limit of stability is defined as $b = V(\underline{\delta}^u, \underline{\omega}^u)$.

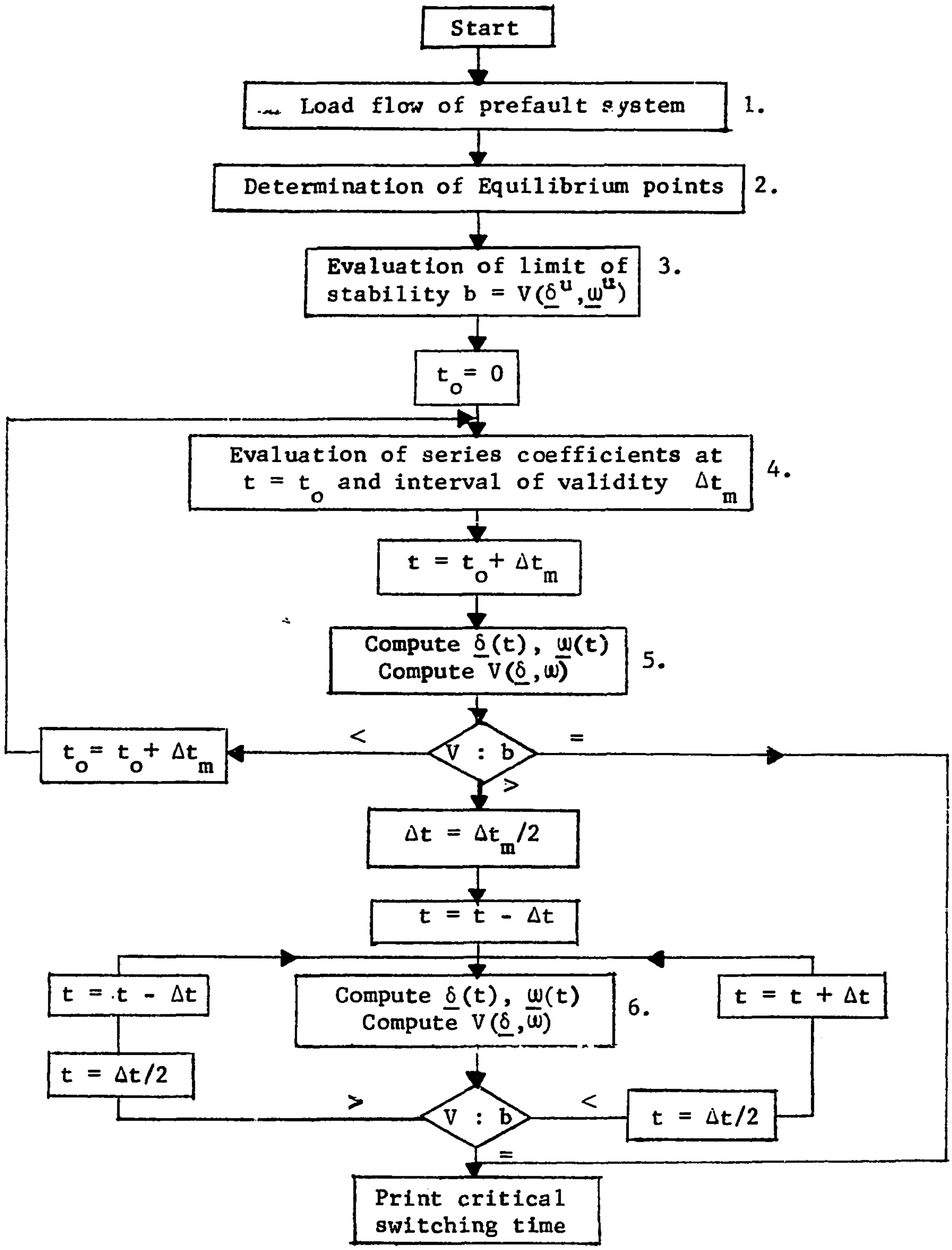

Figure 6.1. Flow chart used in determination of critical switching time using Liapunov's second method

Step 4: Evaluation of Series Coefficients. The solution of the swing equations during the fault condition is represented by a series solution at $t = t_o$ where initially $t_o = 0$. The interval Δt_m at which the truncation error is bounded by some tolerance value is determined to insure accuracy of the method. If the boundary of stability is outside this interval, then the coefficients are recalculated with t_o replaced by $t_o + \Delta t_m$. To check whether the boundary of stability is within the interval $t_o < t < t_o + \Delta t_m$, the variables $\underline{\delta}$ and $\underline{\omega}$ are calculated at the end of this interval, that is, at $t = t_o + \Delta t_m$, and V calculated. If $V(t_o) \leq b \leq V(t_o + \Delta t_m)$ then the boundary is within the time interval Δt_m and so iteration can then proceed to automatically determine the critical switching time.

Step 5: Iteration Process. Having determined the interval $t_o \leq t \leq t_o + \Delta t_m$ a bisection method is used to isolate the point $V = b$. This is done by halving the said interval each time and determining in which half b is located by comparing it with the value of V. When V is at some prescribed tolerance of b then the critical switching time is printed.

6.2. Application to An Actual System

The system used in this study is the combined MERALCO-NPC network described in detail in Appendix 2. There are ten generating stations in the network. Each is represented in this study as a constant voltage behind its transient reactance. Network details as well as prefault conditions obtained from a load flow study of the unfaulted system are given in Appendix 2. This data is necessary to determine the initial conditions of the problem.

Two cases which present two three phase faults at different lines of the network are considered. For each case, the critical clearing time of the faults are estimated using the Liapunov functions of Sections 4 and 5. These are then compared with estimates obtained by point-by-point integration of the swing equations.

Case I

A three phase fault is applied on line 15-43 (Caliraya-Calauan line) close to the Calauan bus represented as Fault 1 on Figure A2.1. The fault is cleared by switching out the faulted line. Hence, the post fault system is the original network less line 15-43.

In the application of Liapunov's theorems the equilibrium points of the post-fault system must be known. An iterative procedure, explained in Section 6.1, was used to obtain these points. The results are given in Table 6.1. The second column gives the stable equilibrium points of the system. In the last column, the "closest" unstable equilibrium state is given. This is assumed to occur when the energy of the system is largely due to the potential energy of the machine most likely to go unstable, that is, the machine with the largest initial acceleration. Using the Liapunov function of section 4 and the procedure outlined in Section 6.1, the critical switching time is estimated to be .379 seconds. By taking into account the energy used in accelerating the center of mass of the system, as given by Equation 5.15, the critical switching obtained is 0.461 seconds. Point-by-point integration for different switching times was made and the critical switching time was found to

Table 6.1. Equilibrium points of the system for Case I

Bus no.	δ_i^s - Stable equilibrium point, degrees	δ_i^u - "Closest" unstable equilibrium point, degrees
1	6.910	9.040
2	17.090	17.090
3	39.577	44.384
4	4.350	6.300
5	21.100	169.683
6	12.906	15.600
7	15.969	19.174
8	6.022	8.816
9	10.002	13.036
10	22.566	49.779

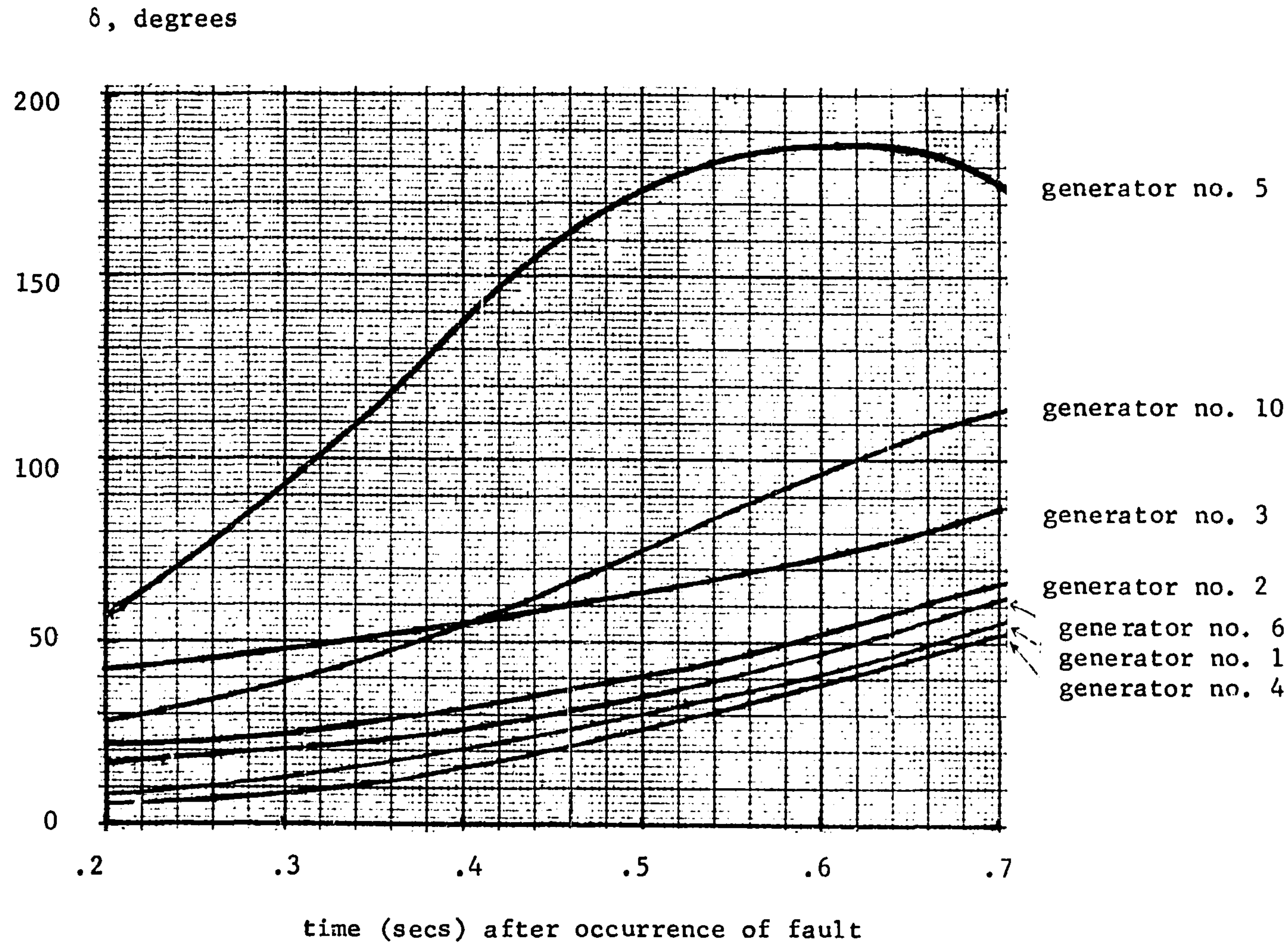

Figure 6.2. Swing curves for Case I with fault cleared in .42 seconds

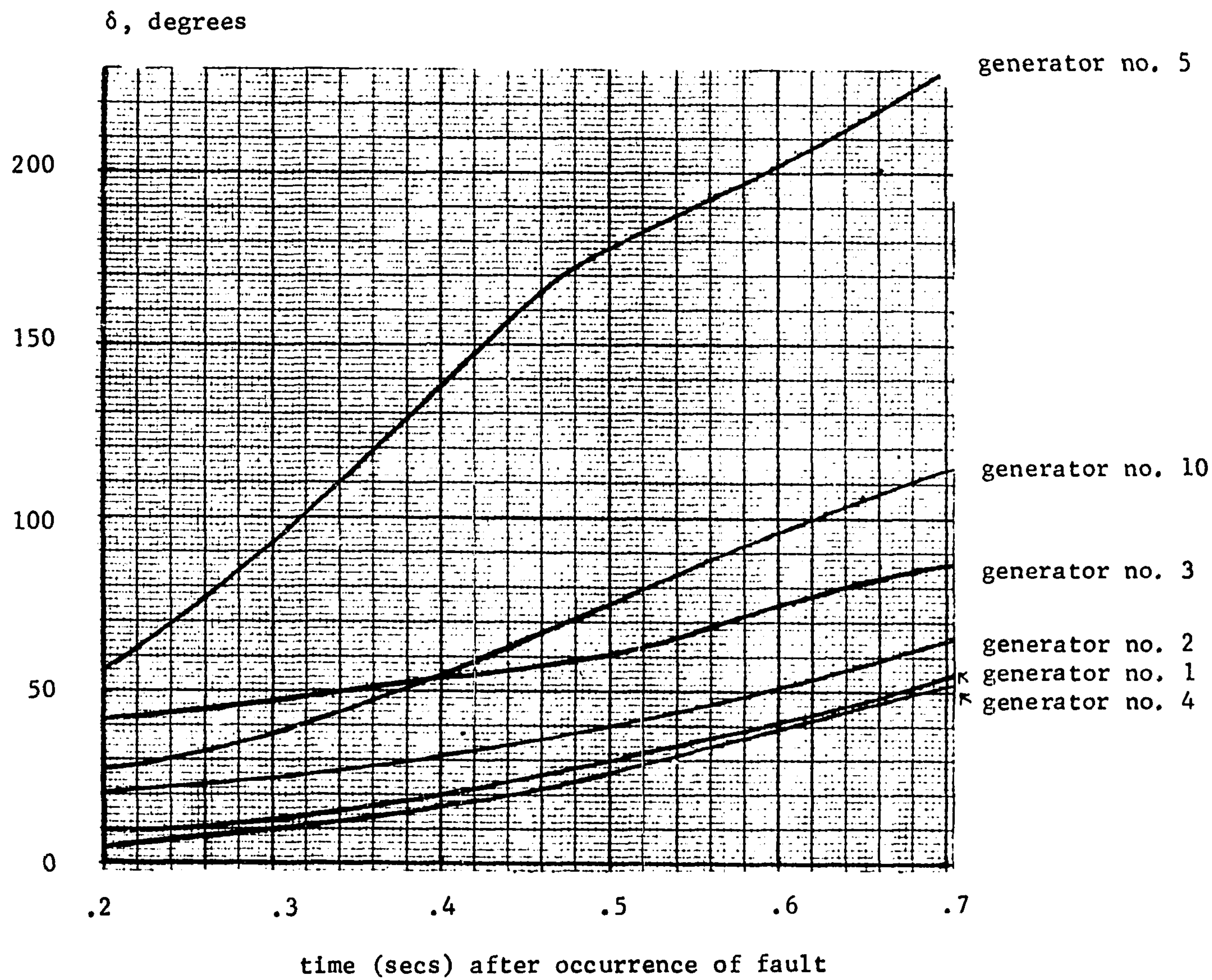

Figure 6.3. Swing curves for Case I with fault cleared in .43 seconds

Table 6.2 Swing data for Case I

	time, secs	Generator angles, degrees										
		δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9	δ_{10}	δ_{5-4}
Fault on	0.00	5.61	17.09	37.66	3.14	19.09	10.99	13.15	3.78	7.41	20.66	15.95
	0.11	6.17	18.18	39.00	3.52	31.23	12.73	15.10	5.26	8.97	22.66	27.71
	0.21	8.45	21.06	42.50	5.31	59.72	16.43	19.44	8.82	12.82	28.55	54.41
	0.31	13.56	25.79	48.07	9.85	97.57	21.09	24.92	13.96	18.37	39.40	87.72
	0.41	21.38	32.62	55.60	17.63	141.41	27.29	31.20	20.66	25.13	55.84	123.78
Fault cleared at .42 secs (stable)	0.42	22.27	33.44	56.46	28.57	146.24	28.06	31.90	21.43	25.87	57.78	127.67
	0.51	31.46	41.83	64.71	28.25	176.30	36.61	39.05	29.54	33.29	76.81	148.05
	0.61	43.46	53.55	75.01	40.71	187.34	49.31	49.72	41.21	43.91	97.91	146.63
	0.71	56.97	67.82	87.05	54.20	173.25	63.90	64.21	55.57	58.18	115.36	119.05
Fault cleared at .43 secs (unstable)	0.43	23.17	34.27	57.34	19.53	151.18	28.86	32.62	22.22	26.63	59.78	131.65
	0.51	31.33	41.83	64.78	28.17	181.52	36.57	39.08	29.51	33.29	76.84	153.35
	0.61	43.21	53.47	75.10	40.53	206.09	49.14	49.67	41.08	43.83	97.76	165.56
	0.71	56.34	67.41	86.97	53.73	232.70	63.31	63.84	55.07	57.79	114.65	178.97

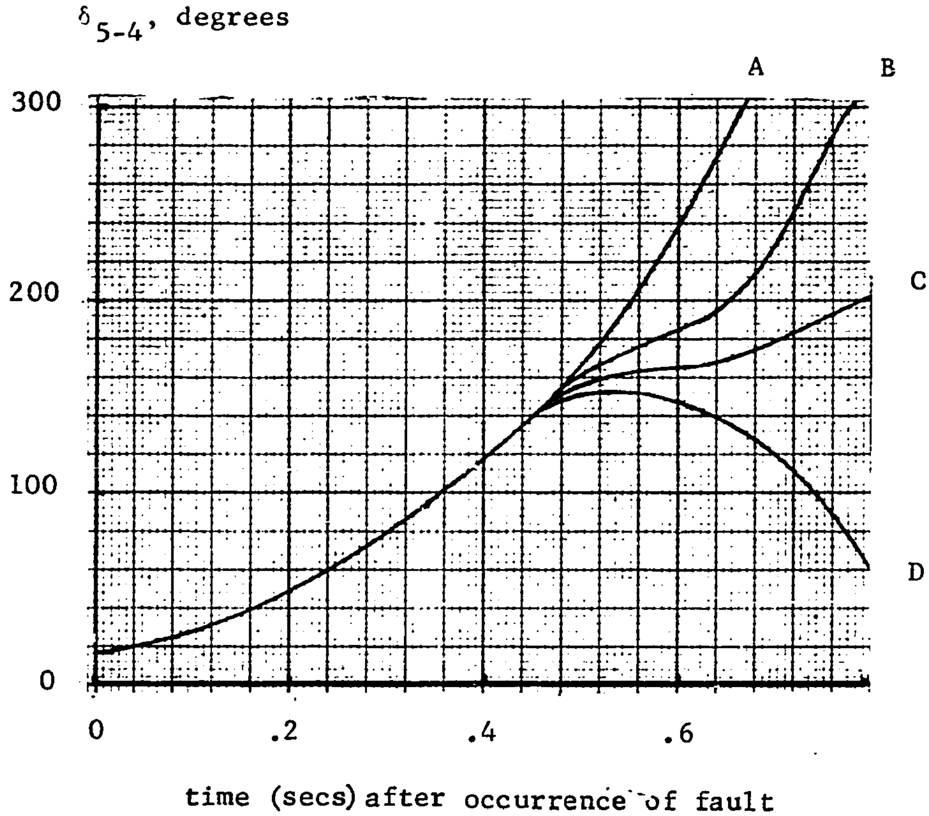

Figure 6.4. Swing curves for Case showing angular difference between generator 5 (Caliraya) and generator 4 (Tegen) for the following switching times:

A. Fault sustained
B. Fault cleared in .44 secs
C. Fault cleared in .43 secs
D. Fault cleared in .42 secs

occur at a little less than .43 seconds. Figure 6.2 shows the swing curves when the fault is cleared in .42 seconds. The curves show that the system is able to recover from the fault. Figure 6.3 shows the swing curves of the system with the fault cleared in .43 seconds which eventually results in instability. The same information is contained in Table 6.2 where the numerical results of the integration of the swing equations are shown for clearing times of .42 and .43 seconds. The critical machines, those with the largest angular difference, are stations Caliraya (generator no. 5) and Tegen (generator no. 4). Their angular difference is plotted versus time as shown in Figure 6.4 for different fault-clearing times. Again, the curves show that the critical switching time occurs at little less than .43 seconds.

The comparison of the results obtained by using the two methods discussed above verifies the ideas proposed in this dissertation. However, there seems to be a discrepancy in the results. The estimate of the critical switching time using the Liapunov function of Section 5 is larger than that obtained by the point-by-point integration. The difference, being 0.03 seconds, is small enough to be negligible for practical purposes. The discrepancy can be attributed to the difference in accuracy of both methods.

The difference in the results obtained by using the Liapunov functions of Section 4 and Section 5 illustrates to a certain extent that the latter is less conservative. This advantage is clearly demonstrated in the next case wherein all the machines more or less swing together.

Case II

A three phase fault occurs on line 12-36-38 (Gardner-Taguig, three-terminal line) close to the Taguig bus designated as Fault 2 on Figure A2.1. The initial conditions are the same as in Case I except for a change in load at bus 45 from 6.1 kw to 61 kw. The fault is cleared by switching out line 12-36-38 at all three terminals. As in the previous case, the post-fault system is the original network less the switched out lines. The equilibrium points of the system are shown in Table 6.3. Point-by-point integration of the swing curves at clearing times of 1 sec. and 1.1 sec. gives the numerical results shown in Table 6.4. Further examination of the results show that in this fault all the machines tend to swing together. Since stability of the system depends only on the angular separation of the machines, not on their individual angles, the swinging of the machines indicates that, from the point of view of stability, this fault is not severe although it gives a big jolt to the system as a whole. This is verified by the results shown in Table 6.4 wherein the critical switching time is between 1 and 1.1 seconds. In Figure 6.5 is plotted the largest angular difference between machines which is that of Montelibano (station no. 3) and Tegen (station no. 4). From Figure 6.5 it is clear that the critical switching time occurs between 1 second (stable) and 1.1 seconds (unstable). It must be noted that a fault clearing time this long is of academic interest only, since system faults would be cleared in a much shorter time. However, this is interesting in the sense that it amplifies the effect of the ideas presented in Section 5. Since all the machines are swinging together, then a great portion of the system energy is used in accelerating the center of mass

Table 6.3. Equilibrium points of the system for Case II

Bus no.	δ_i^s - Stable equilibrium point, degrees	δ_i^u - "Closest" unstable equilibrium point, degrees
1	6.11	8.16
2	17.09	17.09
3	39.83	45.14
4	4.10	6.11
5	20.07	168.72
6	12.43	15.18
7	15.55	18.79
8	5.54	8.38
9	9.56	12.63
10	21.40	33.22

Table 6.4. Swing Case for Case II

	time, secs	Generator angles, degrees										
		δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9	δ_{10}	δ_{5-4}
	0.0	5.61	17.09	37.66	3.14	19.09	10.99	13.15	3.78	7.41	20.66	34.52
	.11	12.17	25.35	46.78	7.38	32.32	21.66	21.53	11.23	14.37	23.90	39.40
	.21	30.12	47.17	71.03	20.63	64.02	47.90	42.80	31.13	33.52	34.72	50.40
	.31	60.63	82.16	110.34	47.13	108.20	85.64	75.54	63.71	65.98	57.77	63.21
	.41	104.94	129.88	164.46	90.75	159.47	131.89	119.79	109.25	112.09	97.43	73.71
Fault	.51	164.01	190.04	233.12	153.06	216.06	186.69	177.29	168.06	171.35	156.09	80.06
on	.61	238.16	262.60	316.01	232.66	279.86	253.14	249.71	240.44	243.35	233.89	83.35
	.71	327.04	247.81	412.86	325.95	355.88	335.08	337.08	326.73	328.66	328.82	86.91
	.81	429.80	446.15	523.51	429.21	450.14	437.89	438.19	427.30	428.61	437.34	94.30
	.91	545.42	558.13	647.95	540.67	566.24	558.70	552.43	542.37	544.00	556.02	107.28
	1.00	659.79	670.89	771.95	648.95	688.43	680.00	667.16	658.17	660.61	670.24	122.86

Table 6.4. (Continued)

	time, secs	Generator angles, degrees δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9	δ_{10}	δ_{5-4}
Fault clear-ed at 1.00 secs (sta-ble)	1.11	809.42	818.94	927.88	797.03	838.07	824.86	818.79	810.33	813.92	819.55	130.85
	1.21	950.82	959.28	1066.36	947.04	962.70	949.34	962.34	952.26	956.86	963.38	129.32
	1.31	1095.92	1105.44	1201.00	1100.78	1096.89	1090.99	1108.31	1095.91	1100.87	1110.99	100.22
	1.41	1246.37	1257.51	1329.52	1249.60	1256.63	1256.34	1255.38	1244.24	1247.83	1260.19	79.92
	1.51	1403.66	1414.74	1450.40	1396.64	1429.51	1423.89	1406.64	1400.54	1401.55	1413.25	53.76
Fault clear-ed at 1.11 secs (un-sta-ble)	1.11	812.42	825.25	938.74	795.17	854.95	839.02	823.89	814.60	817.88	820.21	143.57
	1.21	961.56	973.83	1098.56	948.13	999.05	978.05	976.45	965.39	969.47	968.82	150.43
	1.31	1117.22	1127.88	1260.55	1118.02	1125.06	1111.57	1130.34	1117.91	1123.12	1128.33	142.53
	1.41	1276.25	1286.54	1427.63	1286.95	1266.17	1267.71	1284.80	1273.23	1278.39	1292.06	140.68
	1.51	1437.70	1450.04	1603.17	1443.80	1444.21	1449.94	1443.24	1434.84	1437.69	1454.80	159.37

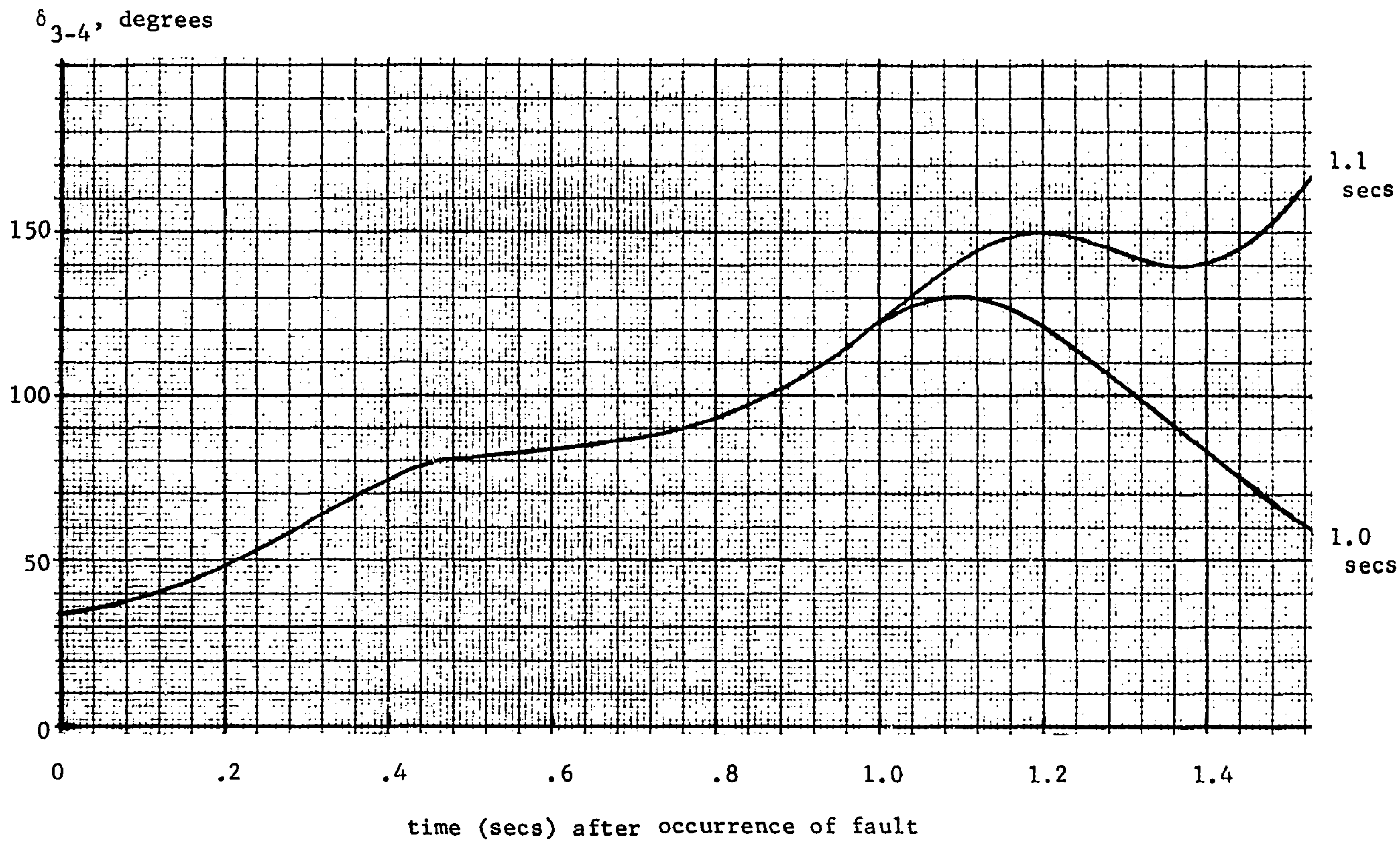

Figure 6.5 Swing curves for Case II showing angular difference between generator 3 (Montelibano) and generator 4 (Tegen) for switching times of 1.0 secs (stable) and 1.1 secs (unstable)

or "inertial center" of the system. It was pointed out in Section 5 that this energy does not affect the stability or instability of the system; all that matters is the movement of the machines relative to one another. Since a great portion of the system energy is due to the movement of the inertial center, its inclusion in the Liapunov function of Section 4 yields very conservative results. This has been the case. This Liapunov function of Section 4 gave, for this fault, an estimate of the critical switching time of .114 seconds. On the other hand, using the Liapunov function of Section 5, which is essentially the total system energy less the kinetic energy of the inertial center, an estimate of .898 seconds is obtained. This is to be compared with the results obtained using numerical integration which yielded an estimate between 1 and 1.1 seconds.

6.3. Conclusions

In Section 6.2, Liapunov's theorems were applied to the study of the stability of a power system. Two different functions were used. The results were compared to those obtained by conventional stability analysis using numerical integration. The index of stability used in this study is the critical clearing time of faults.

The results of Section 6.2 indicate that, in studies involving a power network in which the transfer conductances are negligible, the Liapunov function of Section 4 is suitable. However, for systems with appreciable transfer conductances, the function discussed in Section 4 gives conservative results. This effect is more pronounced in faults leading to a situation in which all the machines tend to swing together. This, as has been pointed out, is due to the fact that the energy due to the movement of the inertial center is included in the energy function of Section 4, although it does not affect the stability of the system. The results also indicate that the function proposed in Section 5 gives a reasonably good estimate of the critical clearing time, even for systems with appreciable transfer conductances. The function proposed in Section 5 is essentially the total energy of the system less the kinetic energy of the inertial center.

Although the results of this study show an improvement in the estimate of the index of stability, further work has to be done to establish the general validity of the method. One of the assumptions used in this study concerns the method of obtaining the unstable equilibrium point "closest" to the stable equilibrium point under consideration. It was assumed that this unstable equilibrium point is in the vicinity of the state of the system where most of the system energy is due to the potential energy of the machine having the largest initial acceleration. Although this sounds logical, a more rigorous justification for this assumption is needed. It is possible that the discrepancy which occurred between the estimate of the clearing time by the Liapunov method of Section 5 and that obtained by numerical integration can be attributed to the fact that the iterative method converges to some unstable equilibrium point other than the closest one. If this is the case, work has to be done to formulate a better method of obtaining the desired equilibrium point.

The above ideas are possible extensions of the present study. In Section 7, more recommendations for future work are listed.

7. RECOMMENDATIONS FOR FUTURE WORK

Although more and more papers are being published on the application of Liapunov's theorems in the field of power system analysis, more work has to be done before a direct method completely replaces the conventional methods being used. The major problems encountered in the application of Liapunov's method to transient stability studies are listed in the introduction. It is sufficient to say that the solutions which have been proposed in this dissertation, although suitable, may not be optimum. Hence, there is still room for work to overcome the difficulties inherent in the method.

One of the major problems is the necessity of a time solution, numerical or otherwise, to obtain the time of approach of the system state up to the boundary of stability. In this study a Taylor series solution of the system equations is used for the fault period. This is possible because the simplified mathematical model of the power system is in such a form that the series coefficients are easy to calculate numerically. This makes it adaptable for computer application. However, the bound on the time interval of validity obtained in Appendix I seems to be too conservative as experience in this study has shown. That is, the series is valid for a much longer interval of time for the same degree of accuracy. Work must be done, therefore, to obtain a better estimate of the bound of the time interval of validity so as to optimize the advantages that can be gained by the use of a series solution.

Other areas of research could be the extension of the proposed concepts to more detailed representation of the power system. One such extension is the incorporation of excitation control. Standard models for representing such equipment are given in the literature. Research on how to account for them in the application of Liapunov's method is needed. The same is true for governor control of turbines. In this study, the time constants of the governor are assumed to be much longer than the time of the transients that occur in the system such that the mechanical input to the generators are practically constant. Since the control equipment affects the dynamic performance of the system, it is important that Liapunov functions used for power system analysis include their effects. Further work is needed in these areas.

Accuracy of the analysis could also be extended by having a more detailed representation of the synchronous machine. In this study, a very simplified model of the synchronous generator is used. Essentially, the generators are represented as constant voltages behind their transient reactances. A more detailed representation is available from the two reaction theory of synchronous machines. However, for larger systems, the dimension of the system equations might be prohibitively large. It would be a big step forward if Liapunov functions using such detailed representation could be developed even for a system comprising one machine against an infinite bus.

The above are some ideas that could be pursued. The references listed in Section 8 provide an unlimited source of ideas for further research.

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9. APPENDIX 1. TAYLOR'S SERIES SOLUTION OF THE POWER SYSTEM EQUATIONS

A1.1. One Machine Connected to an Infinite Bus

Given the following set of differential equations which essentially represents the behavior of a machine connected to an infinite bus

$$\text{(A1.1)} \qquad M\frac{d\omega}{dt} + D\omega = Pm - Pe = M\, f(\delta)$$

$$\frac{d\delta}{dt} = \omega$$

where the function $f(\delta)$ and its derivatives are continuous. It is desired to expand its solution by a Taylor's series, study its convergence and find a bound for the remainder if only n terms of the series are used.

The desired solution will be of the form

$$\text{(A1.2)} \qquad \delta(t) = \delta(o) + \sum_{k=1}^{\infty} \frac{a_k t^k}{k!}$$

$$a_k = \frac{d^k\delta}{dt^k}\bigg|_{t\,=\,0}$$

Expansion of the solution is about $t = 0$ although the discussion to follow is valid for any time $t = t_o$.

For convenience, Equation A1.1 is written in the form

$$\text{(A1.3)} \qquad \frac{d\omega}{dt} = -\frac{D}{M}\omega + f(\delta) = g_2(\omega,\delta)$$

$$\frac{d\delta}{dt} = \omega = g_1(\omega,\delta)$$

so that ω is treated as an independent variable with formal series solutions of the form

$$\text{(A1.4)} \qquad \omega(t) = \omega_o + \sum_{k=1}^{\infty} \frac{b_k}{k!}\, t^k$$

Of course, the b and a coefficients are related if both series converge uniformly and absolutely. In this case differentiation with respect to time would be valid.

Evaluating the coefficients, with all quantities evaluated at $t = 0$

(A1.5) $$a_1 = \frac{d\delta}{dt} = g_1$$

$$a_2 = \frac{d^2\delta}{dt^2} = \frac{\partial g_1}{\partial \omega}\frac{d\omega}{dt} + \frac{\partial g_1}{\partial \delta}\frac{d\delta}{dt}$$

$$a_3 = \frac{d^3\delta}{dt^3} = \frac{\partial^2 g_1}{\partial \omega^2}\left(\frac{d\omega}{dt}\right)^2 + \frac{\partial g_1}{\partial \omega}\frac{d^2\delta}{dt^2}$$

$$\frac{\partial^2 g_1}{\partial \delta^2}\left(\frac{d\delta}{dt}\right)^2 + \frac{\partial g_1}{\partial \delta}\frac{d^2\delta}{dt^2}$$

...
etc.

(A1.6) $$b_1 = \frac{d\omega}{dt} = g_2$$

$$b_2 = \frac{d^2\omega}{dt^2} = \frac{\partial g_1}{\partial \omega}\frac{d\omega}{dt} + \frac{\partial g_2}{\partial \omega}\frac{d^2\omega}{dt^2} + \frac{\partial^2 g_2}{\partial \delta_2}\left(\frac{d\delta}{dt}\right)^2 + \frac{\partial g_2}{\partial \delta}\frac{d^2\delta}{dt^2}$$

$$b_3 = \frac{d^3\omega}{dt^3} = \frac{\partial^2 g_2}{\partial \omega^2}\left(\frac{d\omega}{dt}\right)^2 + \frac{\partial g_2}{\partial \omega}\frac{d^2\omega}{dt^2} + \frac{\partial^2 g_2}{\partial \delta_2}\left(\frac{d\delta}{dt}\right)^2 + \frac{\partial g_2}{\partial \delta}\frac{d^2\delta}{dt^2}$$

...
etc.

To prove convergence, a function $x(t)$ will be used whose convergent series solution dominates the above series. Let two functions $x_1(t)$ and $x_2(t)$ be defined as follows

(A1.7) $$\frac{dx_1}{dt} = \phi_1(x_1, x_2) \qquad x_1(o) = 0$$

$$\frac{dx_2}{dt} = \phi_2(x_1, x_2) \qquad x_2(o) = 0$$

where the functions ϕ_1 and ϕ_2 will later be defined based on certain properties of the system (A1.1).

A formal series solution therefore of the above set of equations are

(A1.8) $$x_1(t) = x_1(o) + \sum_{k=1}^{\infty} \frac{c_k t^k}{k!}$$

$$x_2(t) = x_2(o) + \sum_{k=1}^{\infty} \frac{d_k t^k}{k!}$$

$$c_k = \frac{d^k x_1}{dt^k} \Big|_{t=0}$$

$$d_k = \frac{d^k x_2}{dt^k} \Big|_{t=0}$$

where the coefficients are, with all quantities evaluated at t = 0,

(A1.9) $$c_1 = \frac{dx_1}{dt} = \phi_1$$

$$c_2 = \frac{d^2x_1}{dt^2} = \frac{\partial\phi_1}{\partial x_1} \frac{dx_1}{dt} + \frac{\partial\phi_1}{\partial x_2} \frac{dx_2}{dt}$$

$$c_3 = \frac{d^3x_1}{dt^3} = \frac{\partial^2\phi_1}{\partial x_1^2} \left(\frac{dx_1}{dt^2}\right)^2 + \frac{\partial\phi_1}{\partial x_1} \frac{d^2x_1}{dt^2} + \frac{\partial^2\phi_1}{\partial x_2} \left(\frac{dx_2}{dt}\right)^2 + \frac{\partial\phi_1}{\partial x_2} \frac{d^2x_2}{dt^2}$$

...

etc.

(A1.10) $$d_1 = \frac{dx_2}{dt} = \phi_2$$

$$d_2 = \frac{d^2x_2}{dt^2} = \frac{\partial\phi_2}{\partial x_1} \frac{dx_1}{dt} + \frac{\partial\phi_2}{\partial x_2} \frac{dx_2}{dt}$$

$$d_3 = \frac{d^3x_3}{dt^3} = \frac{\partial\phi_2}{\partial x_1^2} \left(\frac{dx_1}{dt}\right)^2 + \frac{\partial\phi_2}{\partial x_1} \frac{d^2x_1}{dt^2} + \frac{\partial^2\phi_2}{\partial x_2^2} \left(\frac{dx_2}{dt}\right)^2 + \frac{\partial\phi_2}{\partial x_2} \frac{d^2x_2}{dt^2}$$

...

etc.

which are exactly of the same form as Equation A1.5 and A1.6.

If the ϕ functions are now defined to be

(A1.11) $$\phi_1 = \phi_2 = F_1 e^{x_1} + F_2 e^{x_2}$$

where the constants F_1 and F_2 are defined to be the bounds for the terms on the right of Equation A1.12. That is,

$$F_1 \geq \begin{cases} |\omega_o| \\ \frac{D}{M}\,|\omega_o| \end{cases} \qquad \text{(A1.12)}$$

$$F_2 \geq \begin{cases} |f(\delta_o)| \\ \left|\frac{\partial^k f}{\partial \delta^k}\right|_{\delta=\delta_o} \end{cases}$$

the power system Equation A1.1 is usually of such form that the above bounds can be obtained. The following relationships therefore apply (with all values evaluated at t = 0)

$$\emptyset_1 > |g_1| \qquad \emptyset_2 > |g_2| \qquad \text{(A1.13)}$$

$$\frac{\partial^k \emptyset_1}{\partial x_1^k} > \left|\frac{\partial^k g_1}{\partial \omega^k}\right| \qquad \frac{\partial^k \emptyset_2}{\partial x_2^k} > \left|\frac{\partial^k g_2}{\partial \omega^k}\right|$$

$$\frac{\partial^k \emptyset_1}{\partial x_2^k} > \left|\frac{\partial^k g_1}{\partial \delta^k}\right| \qquad \frac{\partial^k \emptyset_2}{\partial x_2^k} > \left|\frac{\partial^k g_2}{\partial \delta^k}\right|$$

Given the above relationships, and from Equations A1.5, A1.6, A1.9 and A1.10, then

$$|a_k| < d_k \qquad \text{(A1.14)}$$

$$|b_k| < c_k$$

which implies that the coefficients of the series of the x functions dominates those of δ and ω. What is left then is to prove that the series for the x functions converge.

From Equations A1.7 and A1.11, the equations governing x_1 and x_2

becomes

$$\text{(A1.14)} \qquad \frac{dx_1}{dt} = F_1 e^{x_1} + F_2 e^{x_2} \qquad x_1(o) = 0$$

$$\frac{dx_2}{dt} = F_1 e^{x_1} + F_2 e^{x_2} \qquad x_2(o) = 0$$

which are identical and hence implies $x_1 = x_2$. Now let

$$\text{(A1.15)} \qquad x = x_1(t) = x_2(t)$$

$$F = F_1 + F_2$$

thus

$$\text{(A1.16)} \qquad \frac{dx}{dt} = F\, e^{x} \qquad x(o) = 0$$

which has a solution

$$\text{(A1.17)} \qquad x = -\ln(1 - \frac{t}{T}) \qquad t < T$$

$$T = \frac{1}{F}$$

which can be represented by a series

$$\text{(A1.18)} \qquad x(t) = x(o) + \sum_{k=1}^{\infty} \frac{1}{k} (\frac{t}{T})^k$$

since

$$\text{(A1.19)} \qquad c_k = \frac{d^k x}{dt^k} \Big|_{t=0} = \frac{(k-1)!}{T^k (1-\frac{t}{T})^k} \Big|_{t=0} = \frac{(k-1)!}{T^k}$$

The above series converges uniformly and absolutely in the interval $o < t < T$ and by comparison so does the series for $\delta(t)$.

A1.2. Multimachine Case

The above concepts are extended for the multimachine case by starting with the system equations

$$\text{(A1.20)} \quad \frac{d\omega_i}{dt} = \frac{D_i}{M_i}\,\omega_i + \frac{1}{M_i}\,[P_{mi} - \sum_{\substack{j=1 \\ j\neq i}}^{N} E_i E_j Y_{ij} \cos(\delta_{ij})]$$

$$\frac{d\delta_i}{dt} = \omega_i$$

$$\delta_{ij} = \delta_i - \delta_j \qquad\qquad i = 1, 2, \ldots, N$$

where all symbols are as defined in Section 4.

For the discussion to follow, it is convenient to rewrite Equation A1.20 as follows

$$\text{(A1.21)} \quad \frac{d\omega_i}{dt} = g_i(\omega_i) + \sum_{j=1}^{N} f_{ij}(\delta_{ij}).$$

$$\frac{d\delta_i}{dt} = h_i(\omega_i) + \sum_{j=1}^{N} l_{ij}(\delta_{ij})$$

where

$$\text{(A1.22)} \quad g_i(\omega_i) = -\frac{D_i}{M_i}\,\omega_i$$

$$f_{ij}(\delta_{ij}) = E_i E_j Y_{ij}\,[\cos(\theta_{ij} - \delta^s_{ij}) - \cos(\theta_{ij} - \delta_{ij})]/M_i$$

$$h_i(\omega_i) = \omega_i$$

$$l_{ij}(\delta_{ij}) = 0$$

A series solution of the above system is desired in the form

$$\text{(A1.23)} \quad \delta_i(t) = \delta_i(o) + \sum_{k=1}^{\infty} \frac{a_{ik}t^k}{k!}$$

$$\omega_i(t) = \omega_i(o) + \sum_{k=1}^{\infty} \frac{b_{ik}t^k}{k!}$$

where

$$\text{(A1.24)} \quad a_{ik} = \left.\frac{d^k\delta_i}{dt^k}\right|_{t=0}$$

$$b_{ik} = \left.\frac{d^k\omega_i}{dt^k}\right|_{t=0}$$

The above coefficients can be expressed in terms of the functions defined in Equation A1.22 as follows, with all quantities evaluated at t = 0.

(A1.25) $$a_{i1} = \frac{d\delta_i}{dt} = h_i + \sum_{j=1}^{N} l_{ij}$$

$$a_{i2} = \frac{d^2\delta_i}{dt^2} = \frac{\partial h_i}{\partial \omega_i}\frac{d\omega_i}{dt} + \sum_{j=1}^{N} \frac{\partial l_{ij}}{\partial \delta_{ij}}\frac{d\delta_{ij}}{dt}$$

$$a_{i3} = \frac{d^3\delta_i}{dt^3} = \frac{\partial^2 h_i}{\partial \omega_i^2}\left(\frac{d\omega_i}{dt}\right)^2 + \frac{\partial h_i}{\partial \omega_i}\frac{d^2\omega_i}{dt^2} + \sum_{j=1}^{N} \frac{\partial^2 l_{ij}}{\partial \delta_{ij}^2}\left(\frac{d\delta_{ij}}{dt}\right)^2 + \frac{\partial l_{ij}}{\partial \delta_{ij}}\frac{d^2\delta_{ij}}{dt^2}$$

...

etc.

(A1.26) $$b_{i1} = \frac{d\omega_i}{dt} = g_i + \sum_{j=1}^{N} f_{ij}$$

$$b_{i2} = \frac{d^2\omega_i}{dt^2} = \frac{\partial g_i}{\partial \omega_i}\frac{d\omega_i}{dt} + \sum_{j=1}^{N} \frac{\partial f_{ij}}{\partial \delta_{ij}}\frac{d\delta_{ij}}{dt}$$

$$b_{i3} = \frac{d^3\omega_i}{dt^3} = \frac{\partial^2 g_i}{\partial \omega_i^2}\left(\frac{d\omega_i}{dt}\right)^2 + \frac{\partial g_i}{\partial \omega_i}\left(\frac{d^2\omega_i}{dt^2}\right) + \sum_{j=1}^{N} \frac{\partial^2 f_{ij}}{\partial \delta_{ij}^2}\left(\frac{d\delta_{ij}}{dt}\right)^2 + \frac{\partial f_{ij}}{\partial \delta_{ij}}\frac{d^2\delta_{ij}}{dt^2}$$

...

etc.

As in the one-machine case, the convergence of the above series will be proven by comparing them with the convergent series of the functions $x_i(t)$ defined by

(A1.27) $$\frac{dx_i}{dt} = \alpha_i(y_i) + \sum_{j=1}^{N}\phi_{ij}(2x_j)$$

$$\frac{dy_i}{dt} = \beta_i(y_i) + \sum_{j=1}^{N}\gamma_{ij}(2x_j)$$

$$x_i(o) = 0$$

$$y_i(o) = 0 \qquad\qquad i = 1, 2, \ldots, N$$

where the functions on the right side of the equation will be defined in terms of certain properties of the system A1.20.

A formal series solution of Equation A1.27 is

$$\text{(A1.28)} \qquad x_i(t) = x_i(o) + \sum_{k=1}^{\infty} \frac{c_{ik}t^k}{k!}$$

$$y_i(t) = y_i(o) + \sum_{k=1}^{\infty} \frac{d_{ik}t^k}{k!}$$

$$c_{ik} = \left.\frac{d^k x_i}{dt^k}\right|_{t=0}$$

$$d_{ik} = \left.\frac{d^k y_i}{dt^k}\right|_{t=0}$$

the c and d coefficients will now be expressed in terms of the functions on the right of Equation (A1.27). With all quantities evaluated at t=0

$$\text{(A1.29)} \qquad c_{i1} = \frac{dx_i}{dt} = \alpha_i + \sum_{j=1}^{N} \emptyset_{ij}$$

$$c_{i2} = \frac{d^2x_i}{dt^2} = \frac{\partial d_i}{\partial \omega_i}\frac{d\omega_i}{dt} + \sum_{j=1}^{N} \frac{\partial \emptyset_{ij}}{\partial x_i} 2\frac{dx_i}{dt}$$

...
etc.

$$d_{i1} = \frac{dy_i}{dt} = \beta_i + \sum_{j=1}^{N} \gamma_{ij}$$

$$d_{i2} = \frac{d^2y_i}{dt^2} = \frac{\partial \beta_i}{\partial \omega_i}\frac{d\omega_i}{dt} + \sum_{j=1}^{N} \frac{\partial \gamma_{ij}}{\partial x_j} (2\frac{dx_j}{dt})$$

...
etc.

which are similar to the form of Equations A1.25 and A1.26.

We now define the α, β, γ and $\emptyset$ functions such that

(A1.30) $$\frac{dx_i}{dt} = A\ e^{2y_i} + \sum_{i=1}^{N} F_j e^{2x_j}$$

$$\frac{dy_i}{dt} = A\ e^{2y_i} + \sum_{j=1}^{N} F_j e^{2x_j}$$

$$x_i(o) = y_i(o) = 0$$

that is

$$\alpha_i(y_i) = \beta_i(y_i) = A\ e^{2y_i}$$

$$\emptyset_{ij}(2x_j) = \gamma_{ij}(2x_j) = F_j e^{2x_j}$$

where

(A1.31)

$$A > \begin{cases} |\omega_i(o)| \\ \\ \dfrac{D_i}{M_i}\ |\omega_i(o)| \end{cases}$$

$$F_j > \begin{cases} |f_{ij}|\ \ \delta_{ij} = \delta_{ij}(o) \\ \\ \left|\dfrac{\partial^k f_{ij}}{\partial \delta_{ij}^{\ k}}\right|\ \ \delta_{ij} = \delta_{ij}(o) \end{cases}$$

for all i

From Equation A1.30, we note that

(A1.32) $$\frac{dx_i}{dt} = \frac{dy_i}{dt}$$

hence $x_i(t) = y_i(t)$. Furthermore, all the equations are the same for all i which implies that

(A1.33) $$x_i = x_j \qquad \text{for all i and j}$$

The inequality A1.31 implies that the functions α, β, γ and ϕ dominate the g, f, h and e functions and their partial derivatives. And in view of the fact that

(A1.34) $$\left|\frac{d\delta_i}{dt}\right| < \frac{dx_i}{dt}$$

then

(A1.35) $$\left|\frac{d(\delta_i - \delta_j)}{dt}\right| < \left|\frac{d\delta_i}{dt}\right| + \left|\frac{d\delta_j}{dt}\right| \leq \left(\frac{dx_i}{dt} + \frac{dx_j}{dt}\right) = 2\frac{dx_i}{dt}$$

Comparing Equations A1.25 and A1.26 to Equations A1.29 and A1.30 shows that each term of the latter dominates each corresponding term of the former. This implies that

(A1.36) $$|a_{ik}| < c_{ik}$$

$$|b_{ik}| < d_{ik}$$

which is the result desired. The next step is to prove that the series for the functions $x_i(t)$ and $y_i(t)$ converge. But from Equations A1.32 and A1.33, all the x's and y's are identical. Representing all these values by x(t), Equation A1.30 becomes

(A1.37) $$\frac{dx}{dt} = F\, e^{2x}$$

(A1.38) $$F = A + \sum_{j=1}^{N} F_j$$

$$x(o) = 0$$

which has a solution

(A1.39) $$x(t) = -\ln\left(1 - \frac{t}{T}\right) \qquad t < T$$

which converges uniformly and absolutely for $0 < t < T$. By comparison therefore the series solutions for $\delta_i(t)$ and $\omega_i(t)$ converge uniformly and absolutely at least in the same interval.

If only n terms of the series solutions are used, a bound for the truncation error can be obtained as follows. The series solution for $\delta_i(t)$ is written in the form

$$\text{(A1.40)} \qquad \delta_i(t) = \delta_i(o) + \sum_{k=1}^{N} \frac{a_{ik} t^k}{k!} + R_{in}(t)$$

$$R_{in}(t) = \frac{t^{n+1}}{(n+1)!} \frac{d^{n+1}\delta_i(t_1)}{dt^{n+1}} \text{ where } 0 < t_1 < t$$

From the previous results

$$\text{(A1.41)} \qquad \left|\frac{d^k\delta_i}{dt^k}\right| \le \frac{d^k x}{dt^k} = \frac{(K-1)!}{T^k(1-\frac{t}{T})^k}$$

$$|R_{in}(t)| \le \frac{t^{n+1}}{(n+1)!} \frac{d^{n+1}x(t_1)}{dt^{n+1}}$$

and since x(t) is monotomically increasing, that is $x(t) > x(t_1)$ for $t > t_1$, then

$$\text{(A1.42)} \qquad |R_{in}(t)| \le \frac{t^{n+1}}{(n+1)!} \frac{n!}{T^{n+1}(1-\frac{t}{T})^{n+1}} \le \frac{1}{n+1} \left(\frac{t}{T-t}\right)^{n+1}$$

Thus by fixing the number of terms used, in this work n is taken to be 9, then truncation error can be controlled by controlling the interval of validity of the series. If the time desired is outside this interval, the coefficients are recalculated as many times as necessary at the forward end of the interval of validity until it includes the time desired.

For practical systems, the bound obtained from Equation A1.42 is too restrictive so that the interval of validity can be expanded considerably without introducing appreciable error.

10. APPENDIX 2. DESCRIPTION OF ELECTRICAL SYSTEM USED IN STUDY

The electrical system used to verify the concepts presented in the main body of this thesis is the combined network of the Manila Electric Company (MERALCO) and the National Power Corporation (NPC). The MERALCO is the biggest private utility company in the Philippines. It serves the city of Manila and suburbs with electric power supplied from four main thermal stations. These are (see Figure A2.1) stations Gardner-Snyder, Rockwell, Tegen and Montelibano which are interconnected by 112 KV and 215 KV networks. MERALCO is interconnected with the government owned National Power Corporation (NPC) network which operates hydro-electric stations Angat, Binga, Ambuklao, Caliraya and Botocan together with Bataan, a thermal station. The NPC operates other power stations outside the island of Luzon which, however, are not interconnected with the system under study. The NPC system is interconnected to the MERALCO network by 215 KV lines. The combined network is essentially a 10-machine system.

The data on the stations with each one reduced to an equivalent one-generator are shown in Table A2.1, all based on 100 MVA, the system base. Data on the lines interconnecting the stations of MERALCO and NPC are given in Table A2.2 and with each line identified by its end buses in Figure A2.1. Table A2.3 gives the result of a load flow study. The constraints on the generation and loads were provided by the power companies. Using the results of the load flow study, the initial conditions for the stability study are computed using the concept that the generators can be represented as constant voltage sources behind their transient reactances. These initial conditions are given in Table A2.4.

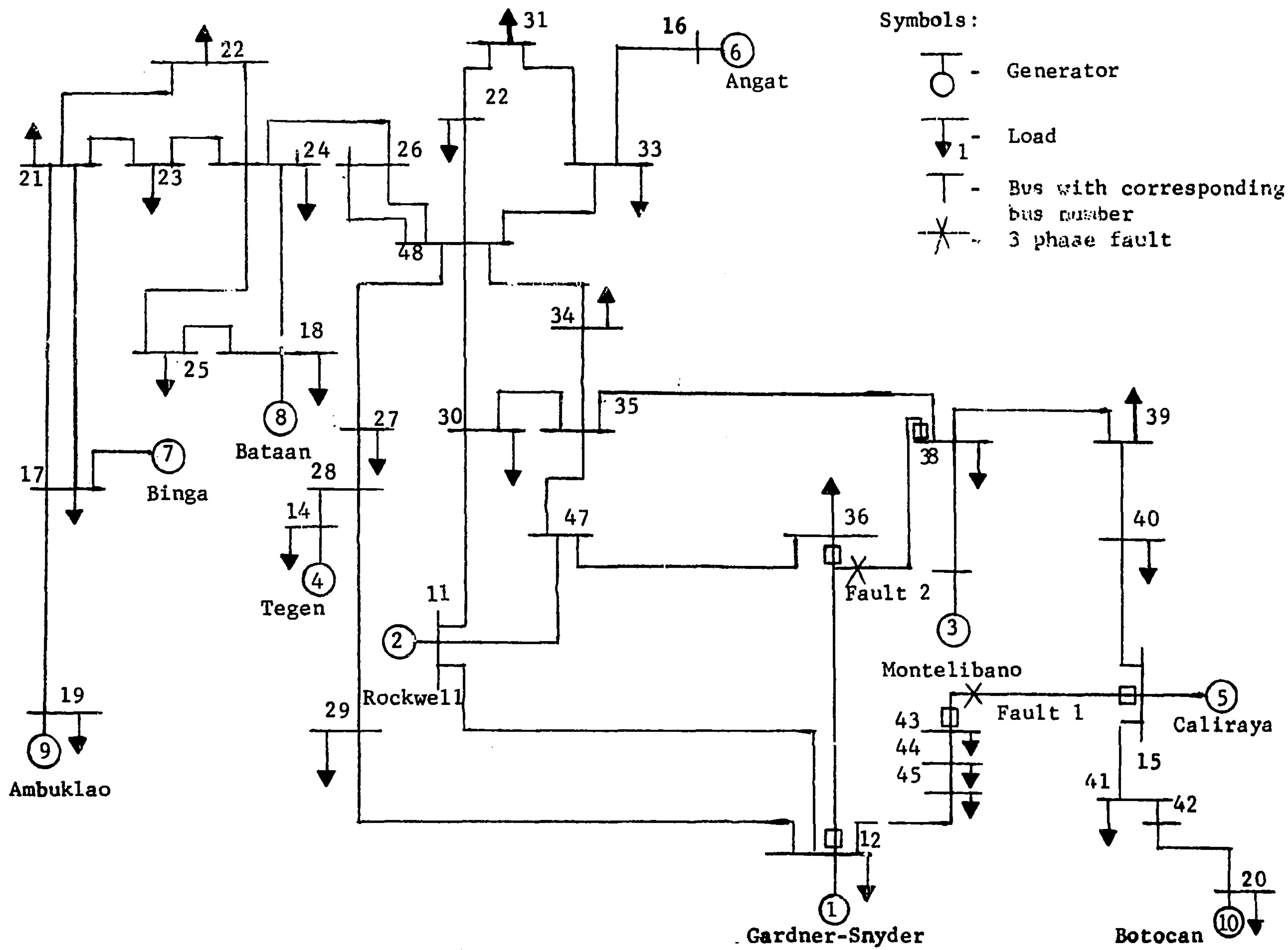

Figure A2.1. Combined MERALCO-NPC electrical system

Case I of Section 6.2 considers a three phase fault on line 15-43 (Caliraya-Calauan line) close to the Calauan bus designated as Fault 1 in Figure A2.1. The fault is then cleared by switching out the faulted line. Case II considers a three phase fault on three-terminal line 12-36-38 (Taguig-Taytay line) close to the Taguig bus. The fault is cleared by switching out the three-terminal line 12-36-38.

Detailed discussion of the above cases is given in Section 6.2.

Table A2.1. Station data

Gen. No.	Station	Rating p.u.[a]	X'd p.u.[a]	H[a]	Type
1	Rockwell	2.45	.0940	3.83	Thermal
2	Gardner	10.40	.0312	4.23	Thermal
3	Montelibano	3.70	.0850	4.50	Thermal
4	Tegen	2.56	.1120	3.50	Thermal
5	Caliraya	0.40	.9500	2.90	Hydro
6	Angat	1.11	.2900	3.00	Hydro
7	Binga	0.46	.7370	3.25	Hydro
8	Bataan	0.82	.2820	3.83	Thermal
9	Ambuklao	1.08	.3050	3.00	Hydro
10	Botocan	0.21	1.5900	2.90	Hydro

[a]All data based on 100 MVA.

Table A2.2. Line data

From bus	To bus	Resistance p.u.	Reactance p.u.
11	47	0.0009	0.0074
11	30	0.0048	0.0194
11	12	0.0046	0.0387
12	29	0.0046	0.0387
12	36	0.0038	0.0338
12	45	0.0147	0.0411
12	46	0.0287	0.2807
13	38	0.0128	0.1546
14	28	0.0000	0.0144
15	43	0.0328	0.0922
15	41	0.0871	0.2463
15	40	0.0637	0.2213
16	48	0.0225	0.1191
16	48	0.0225	0.1191
16	33	0.0275	0.1008
17	19	0.0018	0.0110
18	24	0.0125	0.0728
18	25	0.0114	0.0664
19	21	0.0059	0.0366
19	21	0.0059	0.0366
20	42	0.9970	1.2590
21	22	0.0150	0.0923
21	23	0.0154	0.0923
22	24	0.0122	0.0680
23	24	0.0052	0.0309
24	25	0.0105	0.0612
24	26	0.0099	0.0594
24	26	0.0013	0.0623
26	48	0.0000	0.0332
26	48	0.0000	0.0893

Table A2.2. (Continued)

From bus	To bus	Resistance p.u.	Reactance p.u.
27	28	0.0056	0.0203
27	48	0.0056	0.0194
28	29	0.0036	0.0222
30	35	0.0025	0.0215
30	48	0.0063	0.0255
31	32	0.0369	0.0709
31	33	0.0268	0.0618
32	48	0.0129	0.0240
33	48	0.0138	0.0571
34	35	0.0018	0.0158
34	48	0.0038	0.0236
35	38	0.0018	0.0157
35	47	0.0028	0.0244
36	38	0.0023	0.0201
36	47	0.0009	0.0083
38	39	0.0043	0.0375
39	40	0.0026	0.0225
41	42	0.0000	0.4888
43	44	0.0216	0.0607
44	45	0.0224	0.0626

Table A2.3. Results of load flow study

Bus	Voltage p.u.	Angle degrees[a]	Generation		Load	
			P, p.u.[b]	Q, p.u.[b]	P, p.u.[b]	Q, p.u.[b]
11	1.000	0.00	1.099	0.550	1.5194	0.7160
12	1.031	4.00	8.580	2.820	1.3974	0.4210
13	1.074	26.00	3.000	0.970	0.0000	0.0000
14	0.965	-4.40	1.200	0.750	1.9565	0.5265
15	1.019	3.30	0.360	0.180	0.0524	0.0298
16	1.023	-2.30	1.000	0.625	0.0000	0.0000
17	1.025	-5.00	0.500	0.100	0.4100	0.1100
18	1.025	-6.30	0.750	0.494	0.5200	0.1900
19	1.025	-5.00	0.800	0.191	0.3000	0.1600
20	1.040	7.90	0.154	0.000	0.0911	0.0053
21	1.022	--	0.000	0.000	0.2600	0.0200
22	1.014	--	0.000	0.000	0.2300	0.0600
23	1.012	--	0.000	0.000	0.1150	0.0710
24	1.010	--	0.000	0.000	0.6500	0.1900
25	1.012	--	0.000	0.000	0.3700	0.1400
27	0.997	--	0.000	0.000	1.5635	0.7316
29	0.993	--	0.000	0.000	1.1475	0.4148
30	0.990	--	0.000	0.000	1.2940	0.3045
31	0.970	--	0.000	0.000	0.4976	0.1162
32	0.970	--	0.000	0.000	0.8610	0.3663
33	0.992	--	0.000	0.000	0.1849	-0.0320
34	0.989	--	0.000	0.000	0.5155	0.0562
35	0.990	--	0.000	0.000	0.8368	0.2092
36	1.000	--	0.000	0.000	0.4770	0.0497
38	0.990	--	0.000	0.000	0.6128	0.2710
39	0.981	--	0.000	0.000	0.2765	0.1785
40	0.982	--	0.000	0.000	0.2306	0.0840
42	0.991	--	0.000	0.000	0.0682	0.0319

[a]Angles of bus 21 up are not required and are not included.

[b]Data based on 100 MVA.

Table A2.3. (Continued)

Bus	Voltage p.u.	Angle degrees[a]	Generator		Load	
			P, p.u.[b]	Q, p.u.[b]	P, p.u.[b]	Q, p.u.[b]
43	1.013	--	0.000	0.000	0.0128	0.0449
44	1.012	--	0.000	0.000	0.2095	0.1210
45	1.022	--	0.000	0.000	0.0612	0.0380
46	1.005	--	0.000	0.000	0.4249	0.0599

Table A2.4. Pre-fault data at internal bus of generators

Bus no.	Station	Angle	Voltage	Power
1	Rockwell	5.61	1.0568	1.10
2	Gardner	17.09	1.1461	8.58
3	Montelibano	37.66	1.1750	3.00
4	Tegen	3.14	1.0612	1.20
5	Caliraya	19.09	1.2334	0.36
6	Angat	10.99	1.2332	1.00
7	Ambuklao	13.15	1.1543	0.50
8	Bataan	3.78	1.1791	0.75
9	Binga	7.41	1.1077	0.80
10	Botocan	20.66	1.0663	0.15

11. APPENDIX 3: COMPUTER PROGRAM USED TO APPLY LIAPUNOV'S THEOREMS TO TRANSIENT STABILITY PROBLEMS

List of subroutines

INPU: This subroutine reads in the initial conditions of the study as obtained from a load flow study. This is included in step 1 of the flow chart of Figure 6.1.

BUSIN: This subroutine reads in the admittance matrix of the system reduced to the internal buses of the machines. This is included in step 1 of the flow chart of Figure 6.1.

EQUIL: This subroutine locates the pertinent equilibrium points of the system by performing a steady state solution of the post fault equations. This corresponds to step 2 of the flow chart of Figure 6.1.

LIAP(DELT,V): This subroutine calculates the value of the Liapunov function at any state of the system. This is used in steps 3, 5, 6 of the flow chart of Figure 6.1.

SERIS(TM): This subroutine evaluates the series coefficients of the time solution during fault conditions. The time interval of validity of the series is also determined. This would correspond to step 4 of the flow chart of Figure 6.1.

VALU(T): This subroutine evaluates δ_i and ω_i numerically at any given time during the fault period. This is used in steps 5 and 6 of the flow chart of Figure 6.1.

```
C MAIN PROGRAM
      DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
      DIMENSION PF(10,10),DUST(10),A(10,11)
      COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
      COMMON IREF,RADIN,VCON,VDOT,IOUT,DM
      VDOT=0.0
      ITER=1
      CALL INPU
      IOUT=-1
      CALL BUSIN
      DO 10 I=1,N
      DO 10 J=1,N
      PF(I,J)=P(I,J)
   10 P(I,J)=0.0
      IOUT=+1
      CALL BUSIN
      DO 60 I=1,N
   60 DUST(I)=DEL(I)
      CALL EQUIL
      IF(IOUT-10) 1000,102,102
 1000 DO 61 I=1,N
   61 DST(I)=DUST(I)
      WRITE(3,210)
  210 FORMAT(' ',//,'STABLE EQUIL POINT',/'  GEN    ANGLE')
  212 FORMAT(' ',I5,F10.3)
      DO 111 I=1,N
  111 WRITE(3,212) I,DST(I)
C CALCULATE MACHINE MOST LIKELY TO GO UNSTABLE
      DO 312 I=1,N
      PFLT=PF(I,I)
      DO 313 J=1,N
      IF(I-J) 314,313,315
  314 II=I
      JJ=J
      GO TO 316
  315 II=J
      JJ=I
```

```
316 ANG=(DEL(I)-DEL(J))*RADIN
    G=COS(ANG)*PF(JJ,II)
    B=SIN(ANG)*PF(II,JJ)
    PFLT=PFLT-G-B
313 CONTINUE
    PFLT=ABS(PFLT/AM(I))
    IF(DUM-PFLT) 317,312,312
317 DUM=PFLT
    IST=I
312 CONTINUE
    DUST(IST)=180.-DUST(IST)
    CALL EQUIL
    IF(IOUT-10) 80,102,102
 80 WRITE(3,81)
 81 FORMAT(' ','UNSTABLE EQUILIBRIUM POINT',//,
   1 1X,'  GEN NO',10X,'ANGLE')
    DO 82 I=1,N
 82 WRITE(3,83) I, DUST(I)
 83 FORMAT(' ',I5,5X,F10.3)
    IOUT=-1
    CALL LIAP(DUST,VMAX)
     WRITE(3,50) VMAX
 50 FORMAT('1','STABILITY LIMIT: VMAX=',F10.5,//,
   1'  ITER  TIME  V     VDOT')
    IOUT=1
    JFLAG=-1
    TO=0.0
 20 CALL SERIS(TM)
    T=TO+TM
 21 X=T-TO
    CALL VALU(X)
    CALL LIAP(DEL,V)
    WRITE(3,100) ITER, T,V
100 FORMAT(' ',I5,3F10.5)
    IF(ITER-30) 101,102,102
101 ITER=ITER+1
    IF(JFLAG) 11,11,24
```

```
 11 IF(VMAX-V) 23,23,25
 25 TO=T
    GO TO 20
 23 JFLAG=1
 24 DUM=V-VMAX
    EPS=ABS(DUM/VMAX)
    IF(EPS-.005) 40,40,26
 26 TM=TM/2.
    IF(DUM) 90,40,91
 90 T=T+TM
    GO TO 21
 91 T=T-TM
    GO TO 21
 40 WRITE(3,41) T
 41 FORMAT('1','TCRITICAL=',F5.3,'SECONDS')
    WRITE(3,51)
 51 FORMAT(' ',//,' GEN NO ANGLE   SPEED')
    DO 52 I=1,N
 52 WRITE(3,53) I,DEL(I),W(I)
 53 FORMAT(' ',I5,2F10.4)
102 STOP
    END
```

```
      SUBROUTINE INPU
      DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
      DIMENSION PF(10,10),DUST(10),A(10,11)
      COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
      COMMON IREF,RADIN,VCON,VDOT,IOUT,DM
      WRITE(3,10)
   10 FORMAT('1','GENERATOR DATA',//,
     1' GEN      PM         VOLT     ANGLE        M           D      RATIN')
      READ(1,1) N
    1 FORMAT(I2)
      DO 2 J=1,N
      READ(1,3) I,PM(I),E(I),DEL(I),AM(I),D(I),RATIN
    3 FORMAT(I5,10F10.6)
    2 AM(I)=AM(I)*RATIN/(3.141593*60.)
      RADIN=3.141593/180.0
C   REFERENCE BUS
      EPS=0.0
      DO 31 I=1,N
      IF(AM(I)-EPS) 31,31,32
   32 EPS=AM(I)
      IREF=I
   31 CONTINUE
C CALCULATE MINIMUM D/M
       DM=1.E10
      DO 40 I=1,N
      DUM=D(I)/AM(I)
      IF(DUM-DM) 41,40,40
   41 DM=DUM
   40 CONTINUE
      DO 12 I=1,N
   12 WRITE(3,11) I,PM(I),E(I),DEL(I),AM(I),D(I),RATIN
   11 FORMAT(' ',I5,6F10.3)
      WRITE(3,20) IREF,DM
   20 FORMAT('0','REFERENCE BUS=',I3,//,'MINIMUM D/M=',F10.5)
      DO 50 I=1,N
   50 W(I)=0.0
      RETURN
```

END

```
      SUBROUTINE BUSIN
      DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
      DIMENSION PF(10,10),DUST(10),A(10,11)
      COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
      COMMON IREF,RADIN,VCON,VDOT,IOUT,DM
      DO 9 I=1,N
      DO 9 J=1,N
    9 P(I,J)=0.0
      K=0
      IF(IOUT) 111,112,112
  111 WRITE(3,113)
  113 FORMAT('1','FAULT DATA')
      GO TO 110
  112 WRITE(3,114)
  114 FORMAT('1','POST FAULT DATA')
  110 WRITE(3,101)
  101 FORMAT('0','BUS DATA',//,
     1'  BUS  BUS'/
     2' FROM   TO   COND      SUSC')
C    NONE OF THE I'S MUAT BE ZERO
   10 READ(1,8) I,J,Y,ANG
    8 FORMAT(2I5,2F10.4)
      WRITE(3,50) I,J,Y,ANG
   50 FORMAT(' ',2I5,2F10.4)
      IF(I) 30,30,20
   30 K=1
      I=-I
   20 IF(J) 26,26,22
   22 IF(I-J) 24,24,23
   24 II=I
      JJ=J
      GO TO 25
   23 II=J
      JJ=I
   25 PJ=E(I)*E(J)
      PR=PJ*Y
      PJ=PJ*ANG
```

```
      P(II,JJ)=-PJ
      P(JJ,II)=-PR
      P(J,J)=P(J,J)+Y
   26 P(I,I)=P(I,I)+Y
   17 IF(K) 10,10,18
   18 DO 19 I=1,N
      P(I,I)=E(I)*E(I)*P(I,I)
   19 P(I,I)=PM(I)-P(I,I)
      RETURN
      END
```

```
      SUBROUTINE SERIS(TM)
      DIMENSION PEC(10,10),PES(10,10)
     DIMENSION M(10)
     DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
     DIMENSION PF(10,10),DUST(10),A(10,11)
     COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
     COMMON TREF,RADIN,VCON,VDOT,IOUT,DM
     REAL M
     EQUIVALENCE (M(1),AM(1))
     DO 7 I=1,N
     DO 7 J=1,N
     PEC(I,J)=0.0
   7 PES(I,J)=0.0
     DO 8 I=1,N
   8 PEC(I,I)=PM(I)-PF(I,I)
     IEND=N-1
     DO 10 I=1,IEND
     JST=I+1
     DO 11 J=JST,N
     ANG=(DEL(I)-DEL(J))*RADIN
     DUMC=COS(ANG)
     DUMS=SIN(ANG)
     PEC(I,J)=PEC(I,J)+PF(I,J)*DUMS+PF(J,I)*DUMC
     PEC(J,I)=PEC(J,I)+PF(J,I)*DUMC-PF(I,J)*DUMS
     PES(I,J)=PES(I,J)+PF(I,J)*DUMC-PF(J,I)*DUMS
  11 PES(J,I)=PES(J,I)+PF(I,J)*DUMC+PF(J,I)*DUMS
  10 CONTINUE
C    COMPUTATION OF SERIES CONSTANTS
  13 DO 20 I=1,N
      A(I,1)=DEL(I)*RADIN
     A(I,2)=W(I)
     A(I,3)=PM(I)
     DO 20 K=4,11
  20 A(I,K)=0.
     DO 40 I=1,N
     DO 30 J=1,N
  29 A2=A(I,2)-A(J,2)
```

```
      A(I,3)=A(I,3)-PEC(I,J)
   30 A(I,4)=A(I,4)-A2*PES(I,J)
      A(I,3)=(A(I,3)-D(I)*A(I,2))/M(I)
   40 A(I,4)=(A(I,4)-D(I)*A(I,3))/M(I)
      DO 60 I=1,N
      DO 50 J=1,N
   49 A2=A(I,2)-A(J,2)
      A3=A(I,3)-A(J,3)
      A4=A(I,4)-A(J,4)
      D2=A2*A2
      D3=D2*A2
      D4=D3*A2
      D5=D4*A2
      D6=D5*A2
      C1=A3*A3
      A(I,5)=A(I,5)+D2*PEC(I,J)-A3*PES(I,J)
      A(I,6)=A(I,6)+(D3-A4)*PES(I,J)+3.*A2*A3*PEC(I,J)
      A(I,7)=A(I,7)+(-D4+3.*C1+4.*A2*A4)*PEC(I,J)+6.*D2*A3*PES(I,J)
      A(I,8)=A(I,8)+(-D5+15.*A2*C1+10.*D2*A4)*PES(I,J)
     1+(-10.*D3*A3+10.*A3*A4)*PEC(I,J)
      A(I,9)=A(I,9)+(D6-45.*D2*C1-20.*D3*A4+10.*A4*A4)*PEC(I,J)+
     2(-15.*D4*A3+15.*C1*A3+60.*A2*A3*A4)*PEC(I,J)
      D6=D6*A2
      A(I,11)=A(I,11)+(-D6*A2+56.*D5*A4+210.*D4*C1-280.*D2*A4*A4
     1-840.*A2*C1*A4-105.*C1*C1)*PEC(I,J)+(28.*D5*A2*A3
     2-560.*D3*A3*A4-420.*D2*C1*A3+280.*A3*A4*A4)*PES(I,J)
   50 A(I,10)=A(I,10)+(D6-105.*D3*C1-35.*D4*A4+70.*A4*A4*A2+105.*C1*A4)
     3*PES(I,J)+(21.*A2*D4*A3-210.*D2*A3*A4-105.*A2*C1*A3)*PEC(I,J)
      A(I,5)=(A(I,5)-D(I)*A(I,4))/M(I)
   60 A(I,6)=(A(I,6)-D(I)*A(I,5))/M(I)
      DO 80 I=1,N
      DO 70 J=1,N
   69 A2=A(I,2)-A(J,2)
      A3=A(I,3)-A(J,3)
      A4=A(I,4)-A(J,4)
      A5=A(I,5)-A(J,5)
      A6=A(I,6)-A(J,6)
```

```
      D2=A2*A2
      D3=D2*A2
      A(I,7)=A(I,7)-A5*PES(I,J)
      A(I,8)=A(I,8)-A6*PES(I,J)+5.*A2*A5*PEC(I,J)
      A(I,9)=A(I,9)+(15.*A3*A5+6.*A2*A6)*PEC(I,J)+15.*D2*A5*PES(I,J)
      A(I,11)=A(I,11)+(-56.*D3*A6-520.*D2*A3*A5+35.*A5*A5+56.*A4*A6)
     1*PEC(I,J)+(168.*A2*A3*A6+280.*A2*A4*A5+210.*A3*A3*A5)*PES(I,J)
   70 A(I,10)=A(I,10)+(105.*A2*A3*A5+21.*D2*A6)*PES(I,J)
     1+(-35.*D3*A5+35.*A4*A5+21.*A3*A6)*PEC(I,J)
      A(I,7)=(A(I,7)-D(I)*A(I,6))/M(I)
   80 A(I,8)=(A(I,8)-D(I)*A(I,7))/M(I)
      DO 100 I=1,N
      DO 90 J=1,N
   89 A2=A(I,2)-A(J,2)
      A3=A(I,3)-A(J,3)
      A7=A(I,7)-A(J,7)
      A8=A(I,8)-A(J,8)
      A(I,11)=A(I,11)+(8.*A2*A8+28.*A3*A7)*PEC(I,J)+(28.*A2*A2*A7)
     1*PES(I,J)
      A3=A(I,7)-A(J,7)
      A4=A(I,8)-A(J,8)
      A(I,9)=A(I,9)-A3*PES(I,J)
   90 A(I,10)=A(I,10)-A4*PES(I,J)+7.*A2*A3*PEC(I,J)
      A(I,9)=(A(I,9)-D(I)*A(I,8))/M(I)
  100 A(I,10)=(A(I,10)-D(I)*A(I,9))/M(I)
      DO 501 I=1,N
      DO 500 J=1,N
      A9=A(I,9)-A(J,9)
  500 A(I,11)=A(I,11)-A9*PES(I,J)
  501 A(I,11)=(A(I,11)-D(I)*A(I,10))/M(I)
      KFAC=1
      DO 110 K=3,11
      KFAC=KFAC*(K-1)
      FAC=KFAC
      DO 110 I=1,N
  110 A(I,K)=A(I,K)/FAC
      FMAX=0.0
```

```
      DO 200 I=1,N
      F1=0.0
      F2=0.0
      DO 201 J=1,N
      IF(I-J) 202,201,202
  202 F1=F1+PES(I,J)
      F2=F2+PEC(I,J)
  201 CONTINUE
      F1=F1/M(I)
      F2=F2/M(I)
      DUM=ABS(F2)
      IF(FMAX-DUM) 203,204,204
  203 FMAX=DUM
  204 F2=PF(I,I)/M(I)-F2
      DUM=ABS(F2)
      IF(FMAX-DUM) 205,206,206
  205 FMAX=DUM
  206 DUM=ABS(F1)
      IF(FMAX-DUM) 207,208,208
  207 FMAX=DUM
  208 DUM=W(I)*W(I)
      IF(FMAX-DUM) 209,200,200
  209 FMAX=DUM
  200 CONTINUE
      TM=1./SQRT(FMAX)/2.25
       RETURN
      END
```

```
      SUBROUTINE VALU(T)
      DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
      DIMENSION PF(10,10),DUST(10),A(10,11)
      COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
      COMMON IREF,RADIN,VCON,VDOT,IOUT,DM
C  CALCULATE THE TIME VALUS OF DELTA AND SPEED
      T1=1.
      DO 10 I=1,N
      DEL(I)=A(I,1)+A(I,2)*T
   10 W(I)=A(I,2)
      DO 20 K=3,11
      FAC=K-1
      T1=T1*T
      T2=T1*T
      DO 20 I=1,N
      W(I)=W(I)+FAC*A(I,K)*T1
      DEL(I)=DEL(I)+A(I,K)*T2
   20 CONTINUE
       DO 30 I=1,N
   30 DEL(I)=DEL(I)/RADIN
      RETURN
      END
```

```
      SUBROUTINE LIAP(DELT,V)
      DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
      DIMENSION PF(10,10),DUST(10),A(10,11)
      COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
      COMMON IREF,RADIN,VCON,VDOT,IOUT,DM
      DIMENSION DELT(10)
C CALCULATE LIAPUNOV CONSTANT
       IF(IOUT) 11,31,31
   11 VCON=0.0
      VDOT=0.0
      DO 100 I=1,N
  100 VDOT=VDOT+AM(I)
      IEND=N-1
      DO 20 I=1,IEND
      JST=I+1
      DO 20 J=JST,N
      ANG=(DST(I)-DST(J))*RADIN
      P(J,I)=P(I,J)*SIN(ANG)
   20 VCON=VCON+P(I,J)*COS(ANG)
C  CALCULATE LIAPUNOV FUNCTION
   31 V=VCON
      WM=0.0
      DO 110 I=1,N
  110 WM=WM+AM(I)*W(I)
      WM=WM/VDOT
      DO 30 I=1,N
      DUM=(DELT(I)-DELT(IREF)+DST(IREF)-DST(I))*RADIN
      Y=W(I)+DM*DUM
      Y=W(I)-WM+DM*DUM
      V=V+AM(I)*Y*Y/2.
      DUMB=DM*(D(I)-AM(I)*DM)/2.
       V=V+DUMB*DUM*DUM
      IF(I-N) 50,39,39
   50 CONTINUE
       JST=I+1
      DO 30 J=JST,N
      ANG=(DELT(I)-DELT(J)-DST(I)+DST(J))*RADIN
```

```
    V=V-P(J,I)*ANG
   ANG=(DELT(I)-DELT(J))*RADIN
30 V=V-P(I,J)*COS(ANG)
39 RETURN
   END
```

```
      SUBROUTINE EQUIL
      DIMENSION P(10,10),DEL(10),W(10),E(10),AM(10),D(10),PM(10),DST(10)
      DIMENSION PF(10,10),DUST(10),A(10,11)
      DIMENSION C(10,10),CE(100),L(10),M(10)
      COMMON P,DEL,W,E,AM,D,PM,DST,PF,DUST,A,N
      COMMON IREF,RADIN,VCON,VDOT,IOUT,DM
      TOL=10.0
      DUM=0.0
C FORM JACOBIAN
      NC=N-1
      ITER=1
  100 DO 10 I=1,N
   10 C(I,I)=0.0
      DO 20 I=1,N
      IF(I-IREF) 2,20,1
    1 IC=I-1
      II=IREF
      JJ=I
      GO TO 3
    2 IC=I
      II=I
      JJ=IREF
    3 ANG=(DUST(I)-DUST(IREF))*RADIN
      G=COS(ANG)
      B=SIN(ANG)
      C(IC,IC)=C(IC,IC)+P(II,JJ)*G-P(JJ,II)*B
      IF(I-N) 4,20,20
    4 JS=I+1
      DO 20 J=JS,N
      IF(J-IREF) 6,20,5
    5 JC=J-1
      GO TO 7
    6 JC=J
    7 C(IC,JC)=0.0
      C(JC,IC)=0.0
      ANG=(DUST(I)-DUST(J))*RADIN
      G=COS(ANG)
```

```
      B=SIN(ANG)
      C(IC,JC)=-P(I,J)*G+P(J,I)*B
      C(JC,IC)=-P(I,J)*G-P(J,I)*B
      C(IC,IC)=C(IC,IC)-C(IC,JC)
      C(JC,JC)=C(JC,JC)-C(JC,IC)
   20 CONTINUE
      DO 110 J=1,NC
      DO 110 I=1,NC
      IE=NC*(J-1)+I
  110 CE(IE)=C(I,J)
      CALL AMINV(CE,NC,DET,L,M)
      DO 111 JJ=1,NC
      J=NC+1-JJ
      DO 111 II=1,NC
      I=NC+1-II
      IE=NC*(J-1)+I
  111 C(I,J)=CE(IE)
C CALCULATE FORCE VECTOR
  200 DO 29 I=1,N
   29 C(I,N)=0.0
      DO 30 I=1,N
      IF(I-IREF) 22,30,21
   22 IC=I
      II=I
      JJ=IREF
      GO TO 23
   21 II=IREF
      JJ=I
      IC=I-1
   23 ANG=(DUST(I)-DUST(IREF))*RADIN
      G=COS(ANG)
      B=SIN(ANG)
      C(IC,N)=P(I,I)+C(IC,N)-P(II,JJ)*B-P(JJ,II)*G
      IF(I-N) 24,30,30
   24 JS=I+1
      DO 30 J=JS,N
      IF(J-IREF) 26,30,25
```

```
   25 JC=J-1
      GO TO 27
   26 JC=J
   27 ANG=(DUST(I)-DUST(J))*RADIN
      G=COS(ANG)
      B=SIN(ANG)
      C(IC,N)=C(IC,N)-P(I,J)*B-P(J,I)*G
       C(JC,N)=C(JC,N)-P(J,I)*G+P(I,J)*B
   30 CONTINUE
      WRITE(3,1000) ITER,(DUST(I),I=1,N)
 1000 FORMAT(' ',I5,10F10.3)
C CALCULATE ANGLE CHANGES
      EPS=0.0
      DO 40 I=1,NC
      ANG=0.0
      DO 31 J=1,NC
   31 ANG=C(I,J)*C(J,N)+ANG
      ANG=ANG/RADIN
      DUM=ABS(ANG)
      IF(DUM-EPS) 160,160,161
  161 EPS=DUM
  160 CONTINUE
      IF(DUM-TOL) 153,32,150
  150 IF(ANG) 151,40,152
  151 ANG=-TOL
      GO TO 32
      IF(EPS-TOL) 162,163,163
  152 ANG=TOL
      GO TO 32
  153 CONTINUE
   32 IF(I-IREF) 36,35,35
   35 II=I+1
      GO TO 37
   36 II=I
   37 DUST(II)=DUST(II)+ANG/2.
   40 CONTINUE
  162 TOL=EPS
```

```
  163 CONTINUE
C CHECK CONVERGENCE OF ITERATION
      IF(ITER-50) 41,90,90
   41 ITER=ITER+1
      EPS=0.0
      DO 146 I=1,NC
      IF(I-IREF) 141,142,142
  141 IC=I
      GO TO 143
  142 IC=I+1
  143 DUM=ABS(C(I,N)/PM(IC))
      IF(EPS-DUM) 144,146,146
  144 EPS=DUM
  146 CONTINUE
      IF(EPS-.05) 80,80,100
   80 CONTINUE
      WRITE(3,84) ITER
   84 FORMAT(' ',//,'ITERATIONS=',I3)
      GO TO 999
   90 WRITE(3,91)
      IOUT=10
   91 FORMAT('1','PROBLEM DOES NOT CONVERGE')
  999 RETURN
      END
```